Defence Research and Development Canada
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# Course Loading Optimization

## BACKGROUND

As of 2014, the Naval Personnel Training Group (NPTG) became responsible for all Royal Canadian Navy (RCN) schools. The initial review conducted by the NPTG staff looked at the staff-to-student ratios, which revealed that there were potential inefficiencies that may have increased the cost of school operations. The NPTG then requested the Operational Research Team to conduct a detailed review of the course loading (defined as the number of students enrolled for a given course) and to propose optimal loading which would ensure that a) the RCN personnel gets the required training, b) the course loading is within the estimated minimum and maximum number of required students for each course, and c) the overhead in terms of staff and resource requirements is minimized. The analysis was conducted in two phases. In Phase 1, the minimum and maximum enrollment limits for each course were combined with the historical data [1] to identify the main inefficiencies, including duplicate and excess courses, and courses with insufficient enrollment. These results suggested restructuring the offered course organization in order to meet the demand while eliminating excess. Then, in Phase 2, the course loading was further optimized with respect to resource utilization while considering constraints in terms of student demand and resource supply.

## MAIN OBJECTIVES

- Analyze the supply and demand relationship and identify optimal number of courses that would meet student expected demand while minimizing course cancellations.
- Utilize historical student course loading to estimate the required number of sessions per course, factoring in the risk of having to add or cancel sessions.
- Combine resource constraints with course loading requirements to optimize the course schedule and resource allocation.
- Outside the scope of this study were: location, cost, human factors, and any examination of the ideal minimum and maximum number of students for individual courses.

## METHODOLOGY

### Estimating required course demand

Under the assumption that the historical demand for students is a valid predictor of future demand, the required course estimate was generated based on 2010-2015 student demand. This is combined with Monte-Carlo Simulation Optimization (MCSO), illustrated in the upper box in Figure 1. Probability distributions were generated on a course-by-course basis, where only non-zero student loading was valid input. Options were selected using *@Risk6* or simple representative

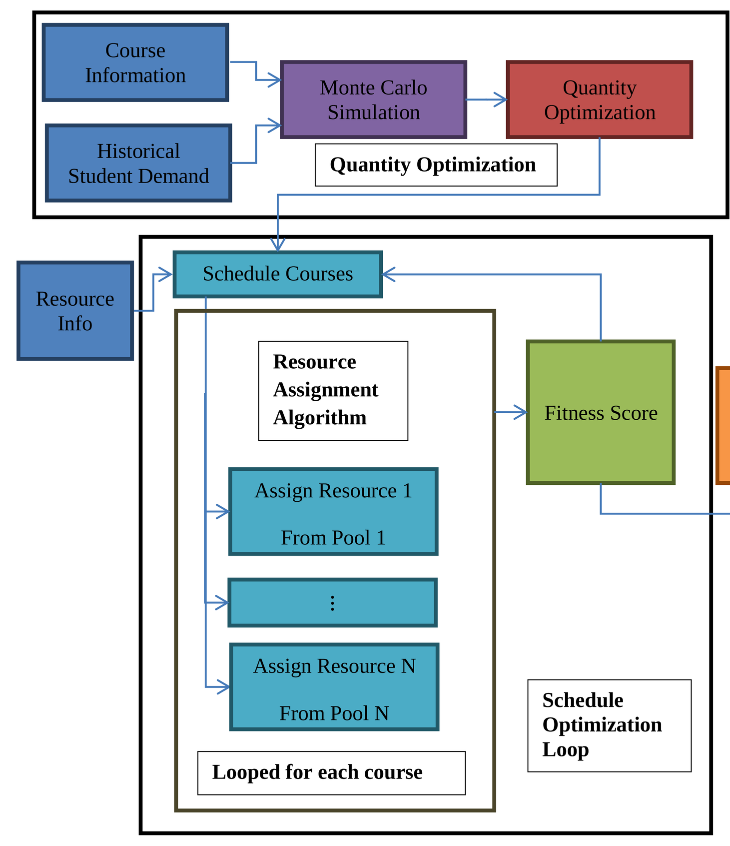



**Figure 1. Representation of the naval schools course schedule algorithm.**

distributions. The expected student load estimates were used to calculate the upper and lower bounds (where the number is expected to increase by one if the remainder meets the minimum course load constraint) for the number of sessions required for each course. To better estimate the risk of having to cancel/add more sessions, the *@Risk6 Optimizer* engine was used to generate the required number of course sessions, with the expected student load probability distributions serving as an input to a MCSO.

Given $n_s$ is the number of students that wish to take course *x*, the optimal number of sessions for each course were calculated such that:

$$\max < f > = \frac{\sum_x (^{n_s}/_{n_{max}})}{x}$$

where *<f>* is the mean fractional capacity, and $n_{max}$ is the maximum number of students allowed in course *x*. Undesirable outcomes are considered as a constraint using the penalty function *c = h + k < a,* where *h* is the number of extra courses that must be added, *k* is the number of cancelled courses, and *a* is a pre-defined constant (oversupply and undersupply of courses are equally undesirable). Essentially, the number of sessions of each course are varied, ensuring that the number falls somewhere between the upper and lower bounds.

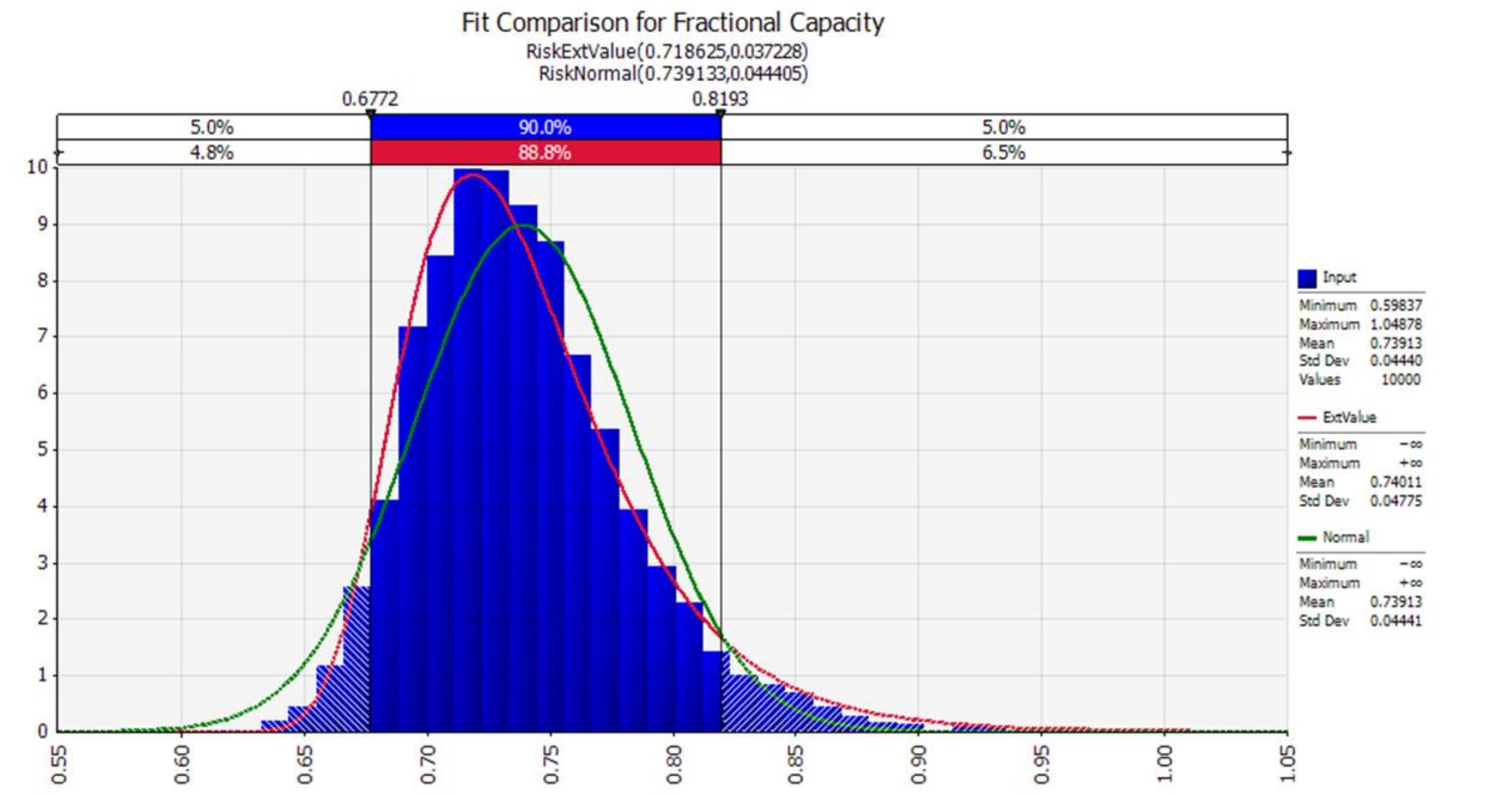
In order to test the approach, the MCSO was run for 5000 iterations of a course *x*. The results for the mean course fractional capacity are shown in Figure 2. From the extreme value distribution the mean fractional capacity, <f>=0.74 +/-0.06 (with ~90% confidence); or, alternatively, there is ~5% probability that the fractional capacity would be below 0.67 (i.e., less than 2/3 of the available seats for this course would be filled).

### Course scheduling and resource assignment

The scheduling optimization problem can be summarized as finding the best choice of start dates for courses, while considering resource assignment limitations, with respect to a fitness function. A weighted sum objective [2] is used, and incorporates cost and schedule conflicts (if any). It was selected because the *Microsoft Excel Solver* add-in is incapable of defining multiple objective problems and this tool selection is currently necessary due to network restrictions. After the fitness function is computed, the optimization loop (lower block in Figure 1) uses *Frontline Systems'* evolutionary algorithm [3] until predefined termination conditions are reached.

A test was developed to determine if the course schedule was feasible; however, the function did not score a completely full schedule (without conflicts) any better than a completely empty schedule. As a result, a new measure was developed for the optimization fitness function, the schedule fractional capacity, $f_{schedule} = t / T$, as a measure of how full the schedule is, where *t* is the total duration of all courses assigned to a resource, and *T* is the total number of available schedule slots. It is

**Figure 2. Example of @RISK6 probability distribution fit for the fractional capacity of course *X*.**




possible for the fractional capacity to yield values greater than one, implying that additional resources would be required to handle the demand.

## RESULTS

A sample problem, based on realistic client data was developed to test the model. A test simulation was run considering 40 courses over a 50 week time span, with 113 resources separated into 9 pools. After running the optimization using the standard Solver, an optimal solution with no schedule conflicts (i.e., a feasible solution) was found, with the fractional capacity $f_{schedule}$ = 0.37. This solution was found within 5 hours on a multi-core computer with 2 Intel i5-4570 processors @3.2GHz.

## CONCLUSIONS

- The course schedule optimization model developed for the RCN performed well for the sample case discussed here; it was able to find a feasible solution for a limited scope problem (in terms of the number of courses and resources).
- The resource assignment algorithm can potentially reduce the problem size by hundreds of orders of magnitude by eliminating the combinatoric nature of resource selection.
- The main limitation of the existing model is the maximum of 200 courses that can possibly be scheduled by the optimization engine. This is due to the 200 variable limit of the *Microsoft Excel Solver* add-in [4].
- The algorithm also exhibits inefficiencies in cases where resources have few course assignments.
- Future work could include development of a more complex objective function, moving away from the weighted sum approach in order to approximate the non-dominated front [2], as well as inclusion of non-critical teaching resources, resource locations and cost.

## MAIN REFERENCES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AUTHOR: Ms. C. Eisler, Dr. P. Dobias, Mr. O. Lu
*DRDC Centre for Operational Research and Analysis*


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