



**COMPARING OBSERVATION  
SATELLITE CONSTELLATIONS – A  
NOVEL APPROACH USING BINARY  
INTEGER PROGRAMMING**

**ISMOR  
July 2015**

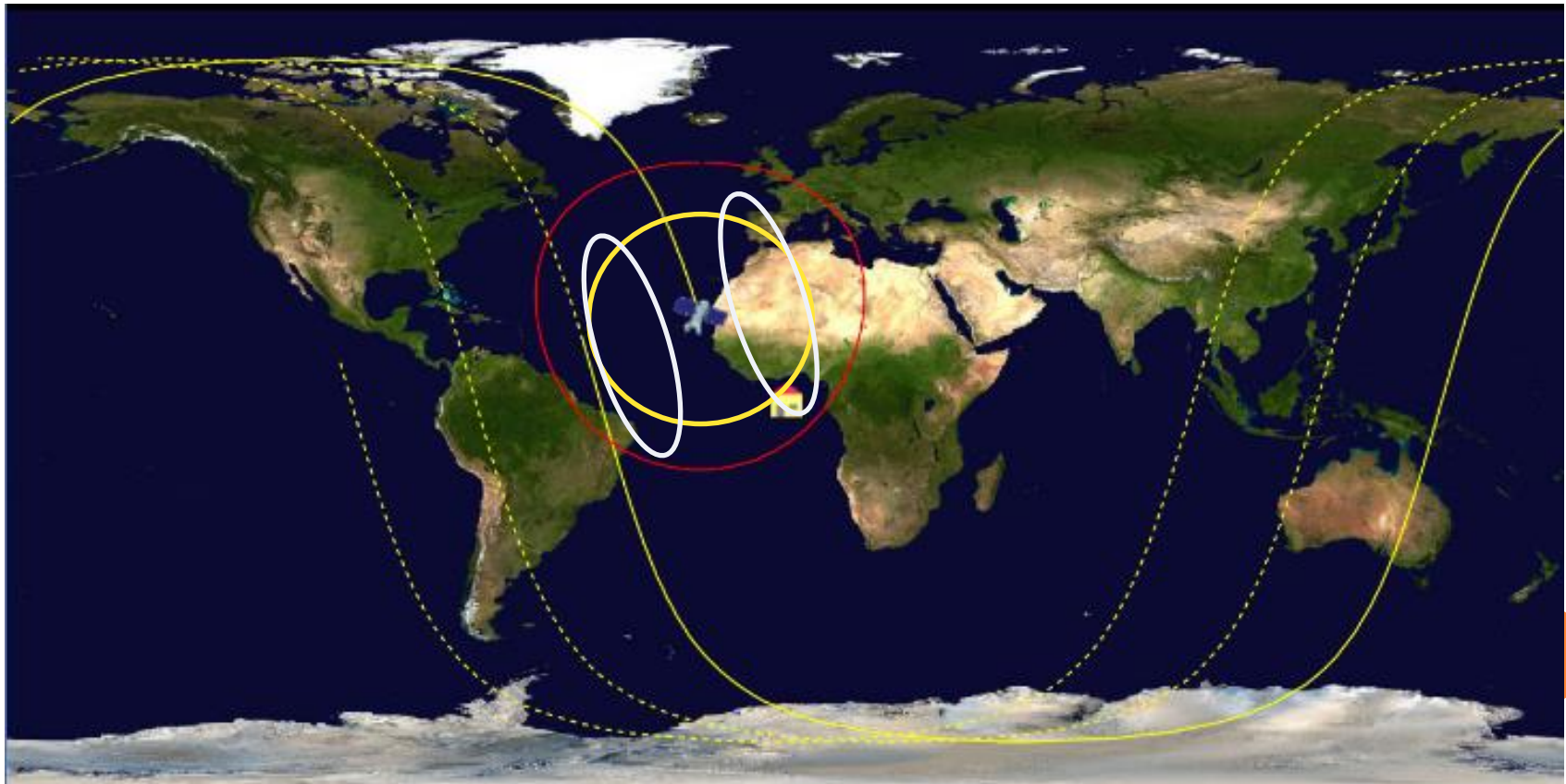
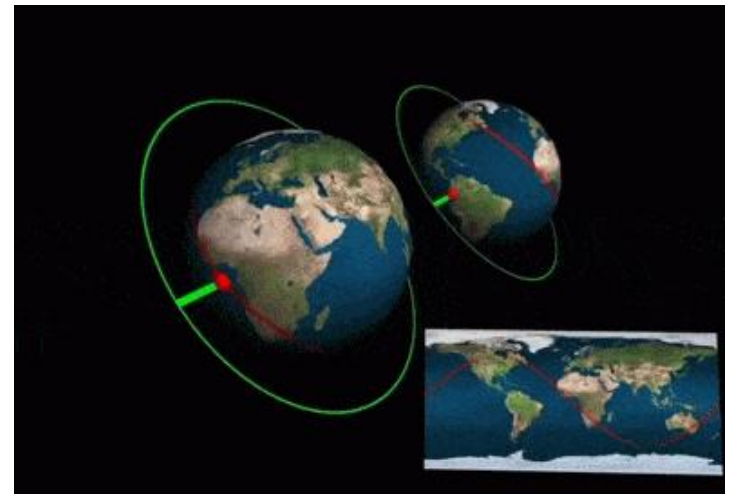
# INTRODUCTION

- The last decade has seen major development of Satellite capabilities:
  - Better resolution
  - Larger coverage
  - Commercially available Multi spectral and SAR images
  - Reduction of costs etc.
- Growing reliance on satellite images (where once other options were necessary)



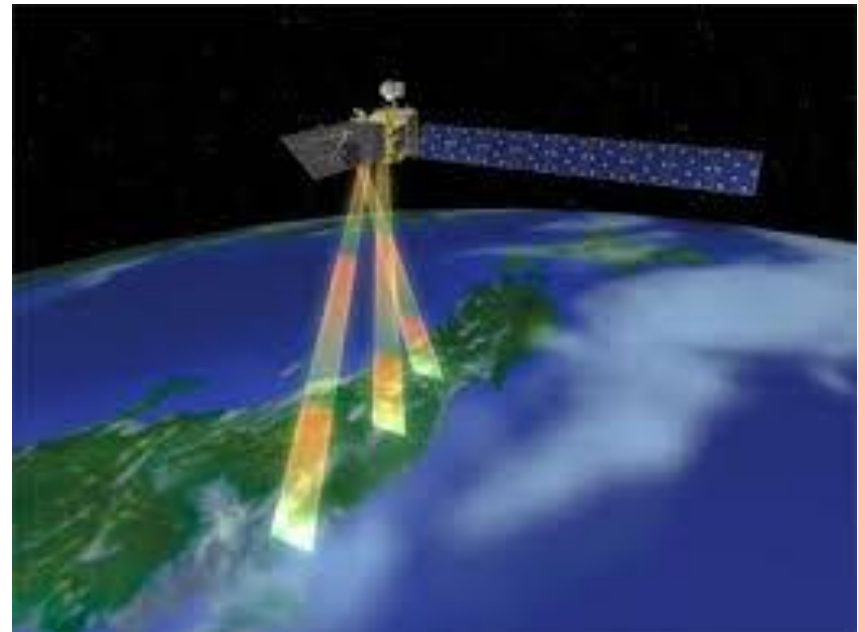
# INTRODUCTION (2)

- Sinusoidal ground track
- Sensors
- Imaging trade-off



# OBJECTIVE

- A quantitative comparison of 3-satellite constellation vs. 2-satellite constellation
  - By assessing the imagery abilities of each satellite constellation



# METHODOLOGY

## Requirements

Identify different satellite image consumers

Define primary requirements (with relevant consumers)

Map crucial requirement parameters

Build requirement DB according to relevant parameters

## Constellations

Create satellite opportunity DB

Analyze satellite DB vs. requirement DB (allocation problem)

Provide recommendations

# SATELLITE OPPORTUNITY DB

Location	Satellite	Time stamp	Resolution	Pass duration
20° latitude	Sat.1 (EO)	Jan. 01 2013 00:12:16	MAX	46s
20° latitude	Sat.2 (SAR)	Jan. 01 2013 03:25:08	Sub 3m	54s
20° latitude	Sat.1 (EO)	Jan. 01 2013 05:07:38	Sub 3m	10s
20° latitude	Sat.1 (EO)	Jan. 01 2013 12:56:20	MAX	72s
20° latitude	Sat.2 (SAR)	Jan. 01 2013 16:32:00	MAX	85s
...	...	...	...	...



# REQUIREMENT EXAMPLES

## Ecological requirements



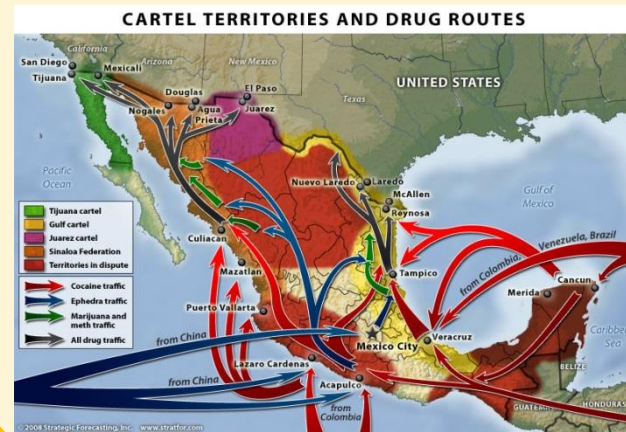
## Natural disaster recovery

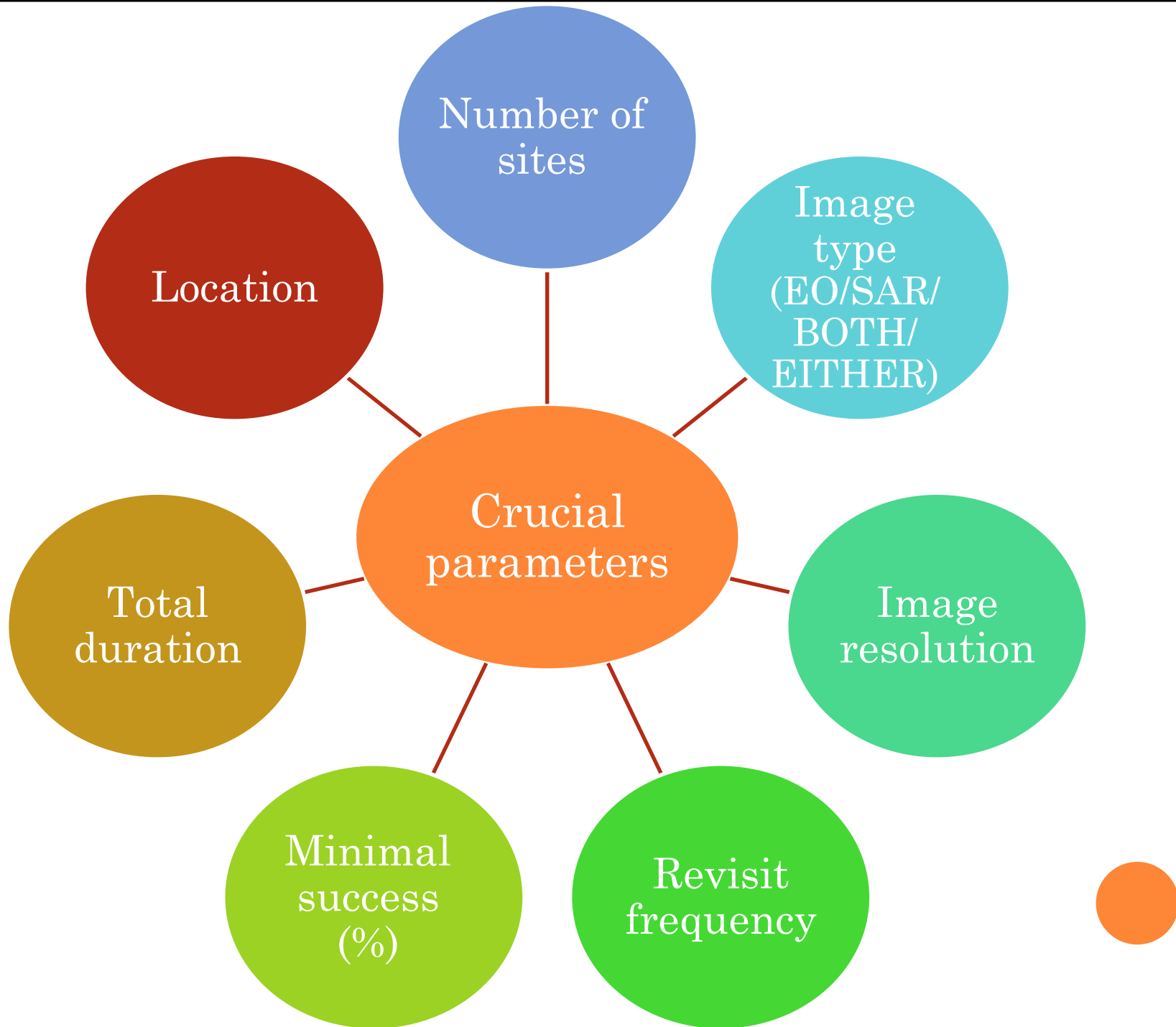


## infrastructure mega-projects



## National civil defense







# REQUIREMENT PARAMETERS EXAMPLES

Requirement	Avalanche detection	Drug trafficking	Animal Migration
Number of sites	1	3	130
Image type	EITHER	EO	EO
Image resolution	max	max	Sub 3 m
Revisit frequency	Daily	Hourly or at least 4 times a day	Weekly
Longest period without image	34 h	4.3 h	10 d
Minimal success	80%	100%	75%
Total duration	6 m	3 d	Year round
Location	Specific (Nepal)	Semi -Specific (Border)	Non -Specific

# APPROACH

- Quantitative measure required → Probability of meeting the requirement → Based on solving an allocation problem

Naïve approach → greedy algorithm

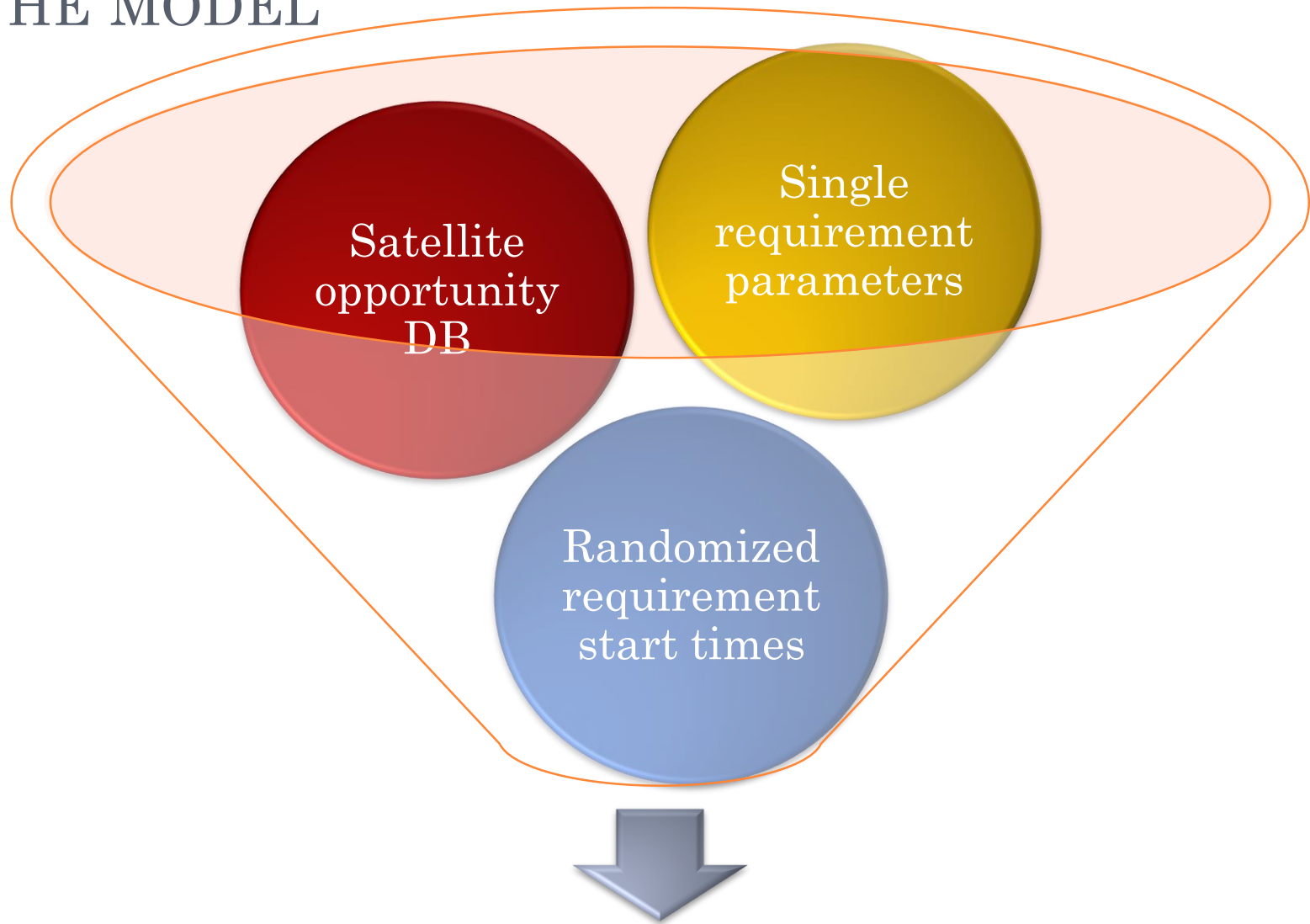
- Fails with mixed image requirement
- Inefficient
- Does not allow estimation of total load

Optimization model

- Binary programming → optimally allocate each opportunity
- Only works for small-medium problem
- Allows load estimation

\*Neither method allows for direct load analysis

# THE MODEL

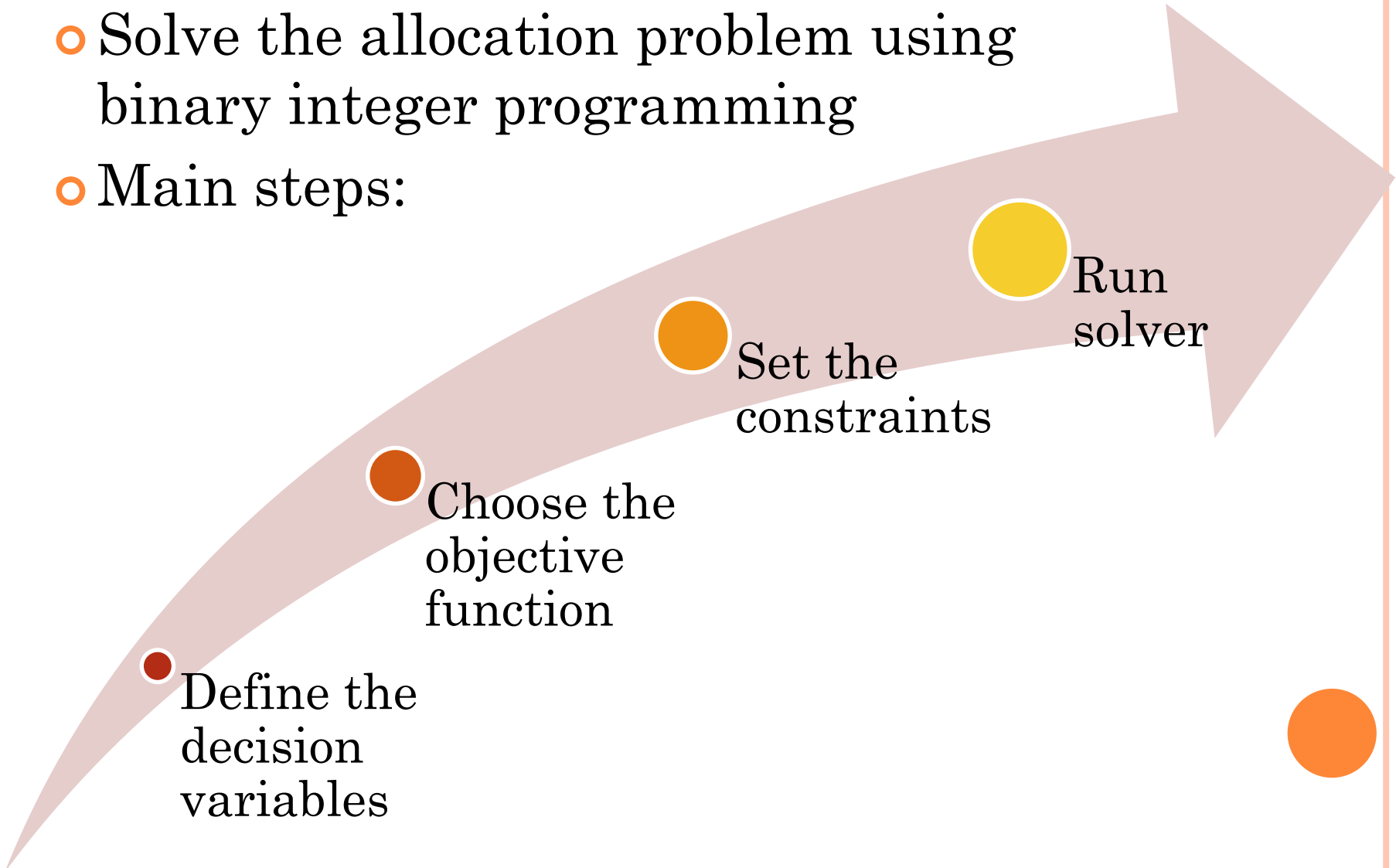


The probability of meeting the requirement  
with each constellation



# PROBLEM STATEMENT

- Solve the allocation problem using binary integer programming
- Main steps:



Define the  
decision  
variables

Choose the  
objective  
function

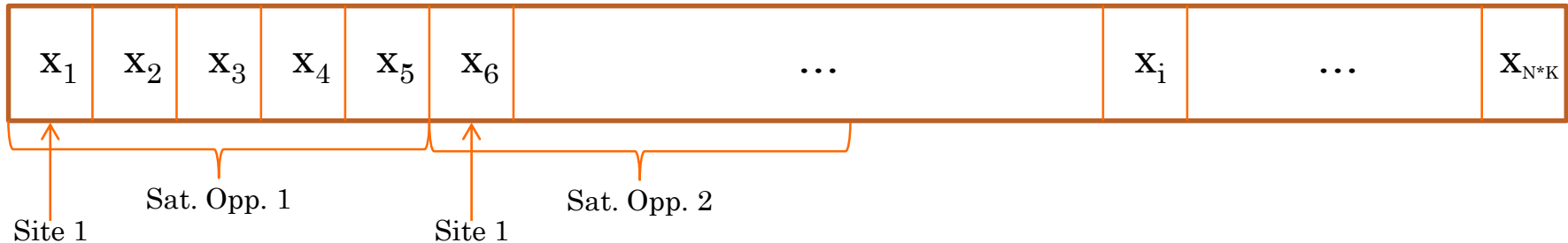
Set the  
constraints

Run  
solver



# MATHEMATICAL FORMULATION — DECISION VARIABLES

- “Allocation opportunity” as the decision variable



- Where:
- $K$  - number of sites in the requirement
- $N$  - number of relevant satellite passes
- $\{x_1 \dots x_{N \cdot K}\}$  - The decision variables vector
  - $x_i$  represents the allocation of pass  $\text{floor}(i/N)$  to site  $\text{mod}(i, K)$



# MATHEMATICAL FORMULATION — OBJECTIVE FUNCTION

- Is there a need for an objective function?
- Our objective → Minimize the total sum of allocated images
  - allows to calculate the load per requirement, hence estimate the total requirement capacity

$$\min(f) = \min\left(\sum_1^{N \cdot K} x_i\right)$$



# MATHEMATICAL FORMULATION — CONSTRAINTS

(1)

- Potential images constraint:
  - Each satellite pass allows for a given # of potential images per area

$$\forall j \in (1, N), \quad \sum_{l=(j-1) \cdot K + 1}^{j \cdot K} x_l \leq n_j$$

- Where:
- $n_j$ - the total number of images for a given area in the  $j$ -th pass

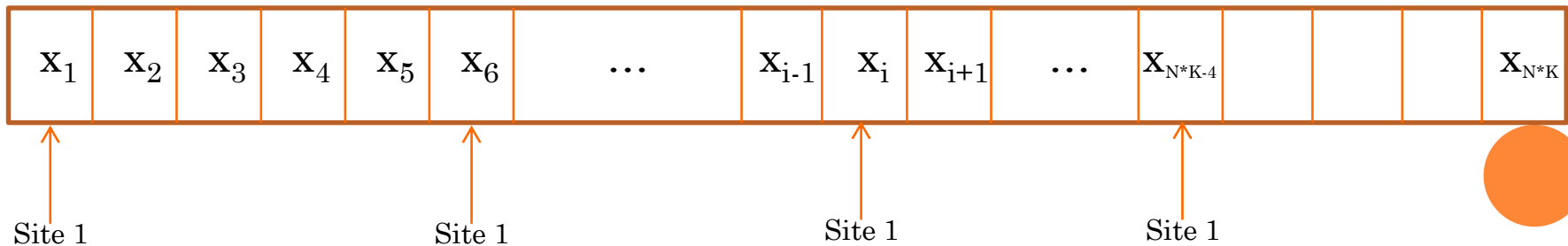


# MATHEMATICAL FORMULATION — CONSTRAINTS

(2)

- Minimal necessary allocations constraint:
  - For each site in the requirement there is a minimal necessary allocation of at least  $p$  images

$$\forall k \in (1, K), \quad \sum_{r=1}^N x_{(r-1) \cdot K + k} \geq p$$




# MATHEMATICAL FORMULATION — CONSTRAINTS

(3)

- Minimal necessary allocations constraint 2:
  - Each site may also contain separate SAR and EO allocation

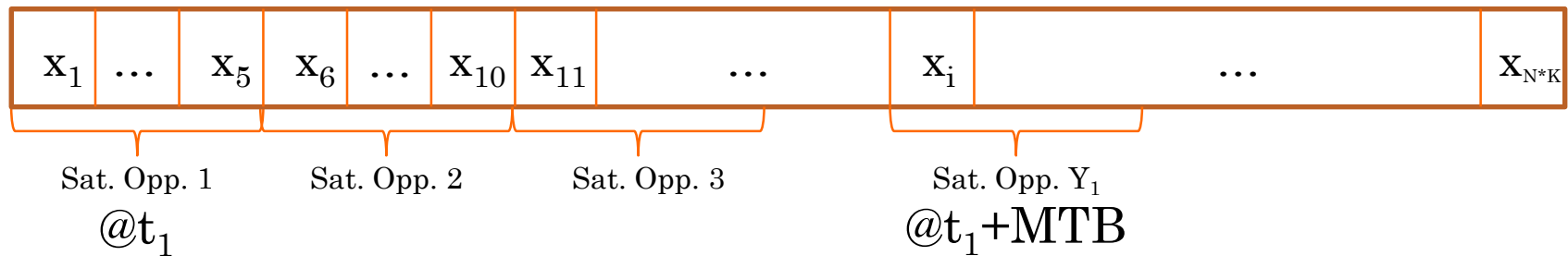
$$\forall k \in (1, K), \forall s \in N_{SAR}, \quad \sum_s x_{(s-1) \cdot K + k} \geq p_{SAR}$$
$$\forall k \in (1, K), \forall q \in N_{EO}, \quad \sum_q x_{(q-1) \cdot K + k} \geq p_{EO}$$

- Where:
  - $p_{SAR}$ - the min # of SAR images;  $p_{EO}$ - the min # of EO images;
  - $N_{SAR}$ - the total SAR capacity in the relevant time period;
  - $N_{EO}$ - the total EO capacity in the relevant time period.
- 

# MATHEMATICAL FORMULATION — CONSTRAINTS

(4)

- Maximum time between allocations constraint:
  - For each requirement there is a maximal time allowed between two allocations

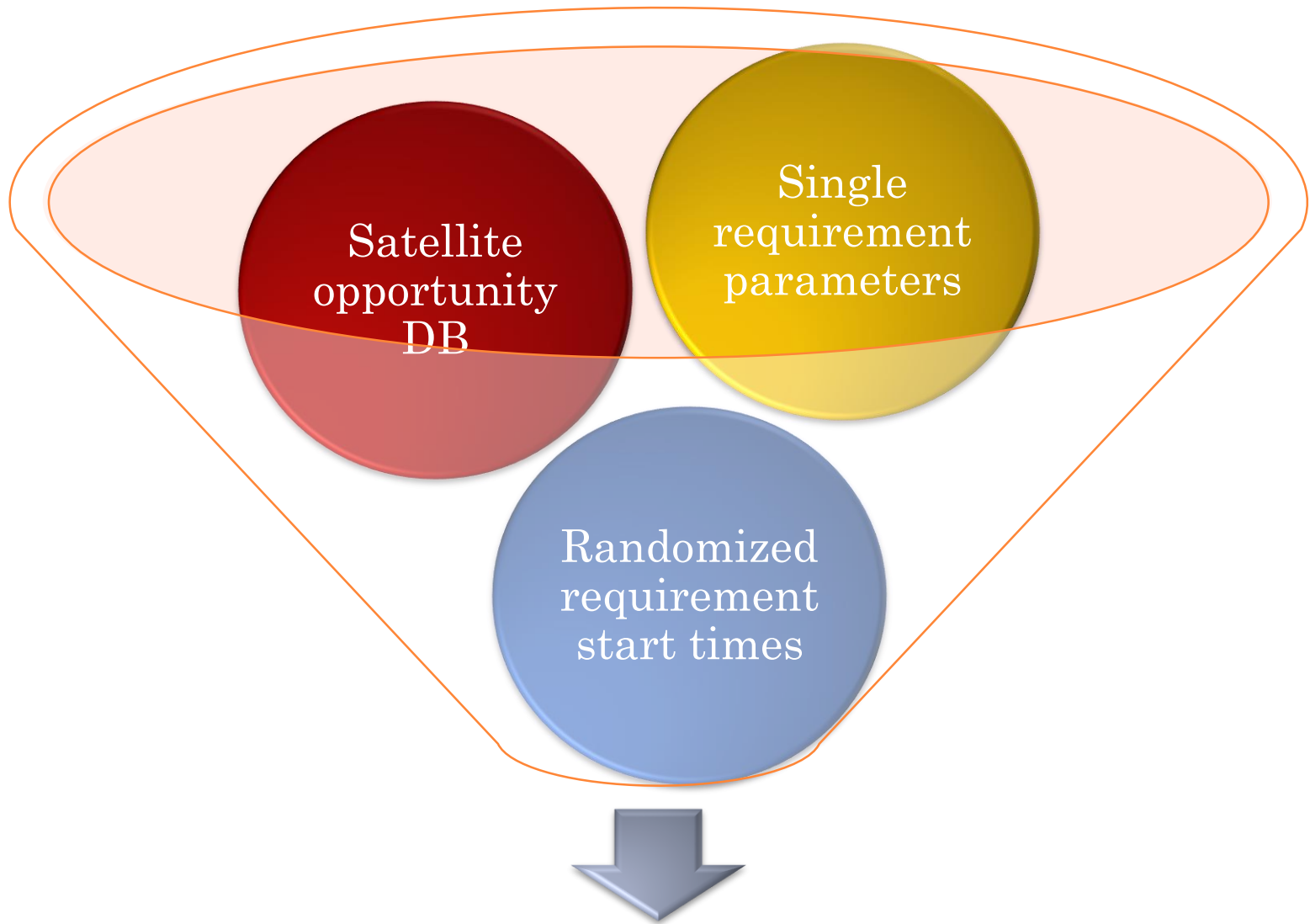


$$\forall n \in N, \quad \forall k \in K, \quad \sum_{t=0}^{Y_n-1} x_{(n+t) \cdot K + k} \geq 1$$

- Where:
- $Y_n$  - the number of passes within the maximal time period from the  $n$ -th pass



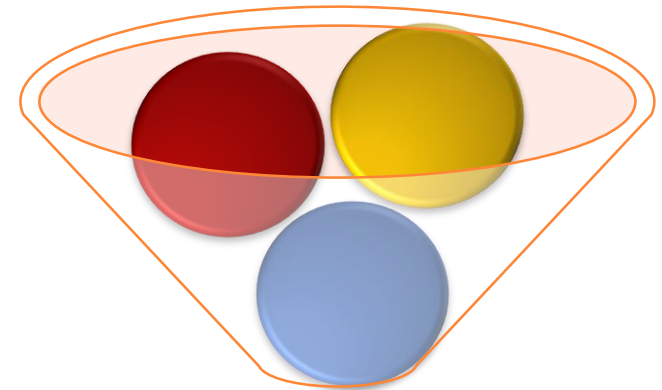




The probability of meeting the requirement  
with each constellation



# SOLVING THE PROBLEM



Probability of meeting the requirement

Start time	Constraint				Conclusion
	1	2	3	4	
Jan 01 2013 02:03:56	✓	✓	✓	✓	✓
May 18 2013 12:02:03	✓	✓	✓	✗	✗
March 30 2013 16:17:15	✓	✓	✗	✓	✗
December 05 2013 01:18:36	✓	✓	✓	✓	✓
May 06 2013 18:03:15	✓	✓	✓	✓	✓
...					
Final conclusion					60%

# FULL RESULTS FOR A SINGLE REQUIREMENT

Few days with insufficient passes cause failure due to max time constraint

Latitude	2-satellite constellation	3-satellite constellation
20°	90%	100%
30°	90%	100%
40°	60%	80%
50°	20%	70%
60°	0%	0%

Multiple days with insufficient passes cause failure due to max time constraint

Limited # of max res passes do not allow for total # of req images

# SUMMARY & CONCLUSIONS

- Our analysis points to a substantial increase in satellite relevance due to technological improvement
- Addition of third satellite improves capabilities but only slightly
- Full analysis contains:
  - Method for combining results from different requirement
  - Detailed load estimation
  - Addition of important qualitative factors
  - Budgetary limitations



THANK YOU FOR YOUR ATTENTION!

