Concentration and Asymmetry in Air Power

Lessons for the defensive employment of small air forces

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What do the **historical data** tell us about the nature of concentration in air combat?

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What can a **historical analysis** tell us about the implications for tactical and operational principles?

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$$\frac{dG}{dt} = -rR$$

Green's **instantaneous** loss-rate is proportional to Red numbers

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and vice versa.

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and vice versa. Divide:

$$\frac{dR}{dG} = \frac{gG}{rR} \qquad \text{or} \qquad rR\,dR = gG\,dG$$

and integrate:

$$\frac{1}{2}rR^2 = \frac{1}{2}gG^2 + \text{constant}$$

throughout the battle, the **Square Law**.

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suppose we begin with twice as many Reds as Greens, $R_0 = 2G_0$,
but that Greens are three times more effective, $g = 3r$.
Then

$$rR^2 - gG^2 = r(2G_0)^2 - 3rG_0^2 = rG_0^2 > 0,$$

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Concentration is good:
If Red divides its forces, and Green fights each half in turn,
Green wins the first battle, with $\sqrt{2/3} \simeq 80\%$ of G_0 remaining,
Green wins the second battle, with $\sqrt{1/3} \simeq 60\%$ of G_0 remaining.

Generalized scaling laws for air combat

Fit loss-rates to powers of own and enemy numbers:

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is constant, where $\rho = 1 + r_1 - r_2$ and $\gamma = 1 + g_1 - g_2$,
the **exponents**, capture the conditions of battle:

– Green should concentrate its force if $\gamma > 1$, divide if $\gamma < 1$.

– if $\rho > \gamma$ then Green has a defender's advantage, by a factor ρ/γ

Symmetric dynamics: The loss and force ratios

The crucial tactical relationship is

$$\frac{dG}{dR} = \frac{r}{g} \frac{R^{\rho-1}}{G^{\gamma-1}} .$$

If the dynamics are **symmetric**, $\rho = \gamma$, we can ask:

How does the **loss ratio** dG/dR depend on the **force ratio** R/G ?

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Two obvious possibilities are

Lanchester's square law: simple proportionality, $\rho = \gamma = 2$

Lanchester's linear law: no dependence

Symmetric dynamics: The loss and force ratios

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– Col. John Warden, USAF, *The Air Campaign*

Cites a 1970 study of Korea and WW2.

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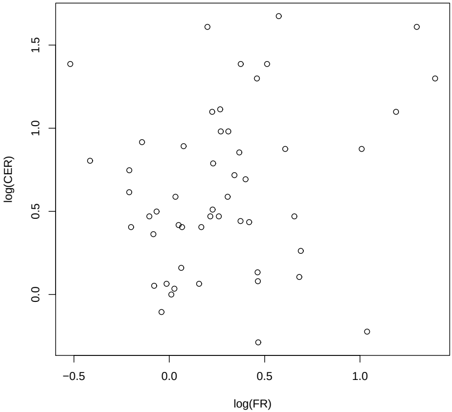
Well, no.

NJM, Is air combat Lanchestrian?, *MORS Phalanx* **44**, no. 4 (2011) 12-14

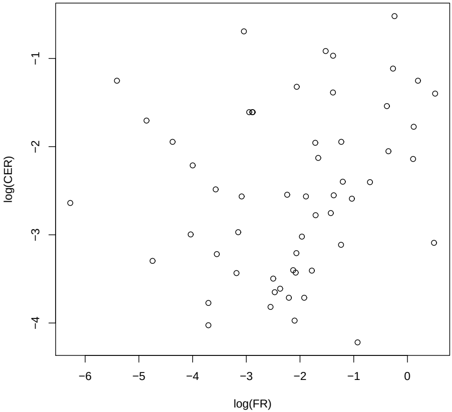
The loss ratio: Battle of Britain



$$\log dG/dR \ \ vs \ \log R/G$$

G=Luftwaffe, R=Royal Air Force

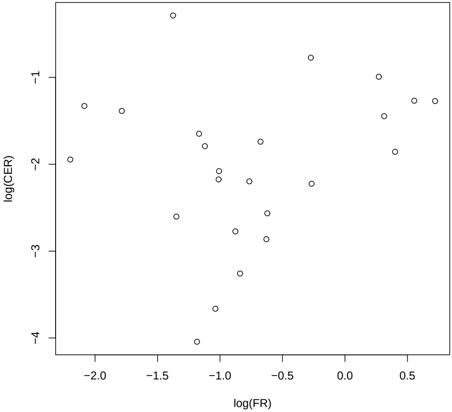
The loss ratio: Pacific air war



$\log dG/dR$ *vs* $\log R/G$

G=Americans, R=Japanese

The loss ratio: Korea



$\log dG/dR$ vs $\log R/G$

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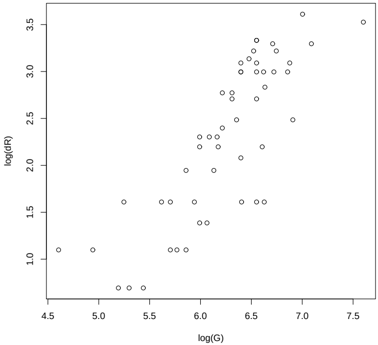
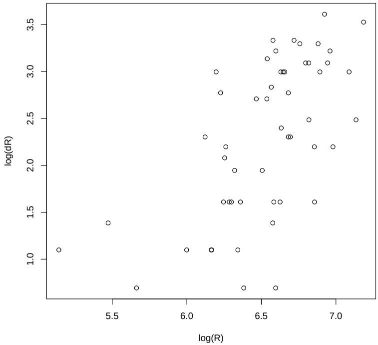
But air combat is **asymmetric**.

Battle of Britain: RAF losses



$\log \delta R$ *vs* $\log G$



$\log \delta R$ *vs* $\log R$

$$\frac{dR}{dt} = -g\,G^{1.12\pm0.17}\,R^{0.18\pm0.25}$$

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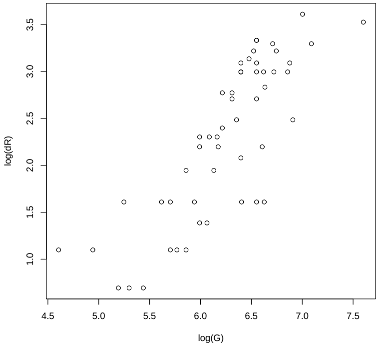
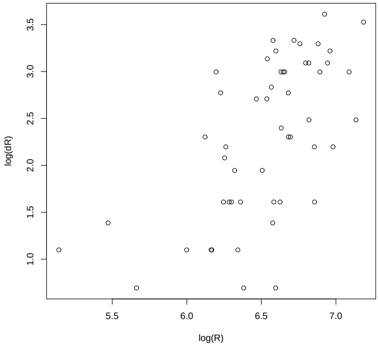


log δR *vs* log G

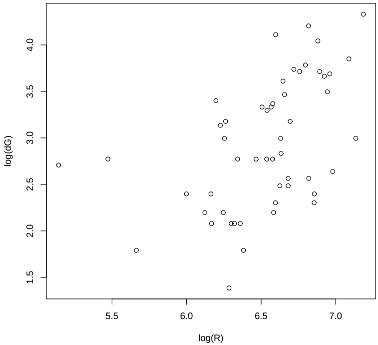


log δR *vs* log R

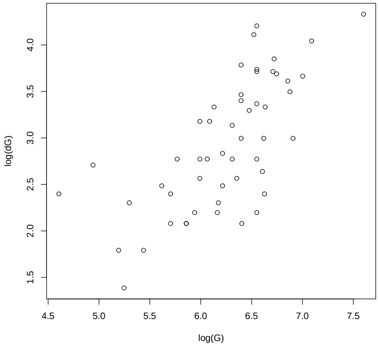
$$\frac{dR}{dt} = -g\,G^{1.12\pm0.17}\,R^{0.18\pm0.25}$$

Hooray for Lanchester!

Battle of Britain: Luftwaffe losses



log δG vs log R



log δG vs log G

$$\frac{dG}{dt} = -rR^{0.00\pm0.25}\,G^{0.86\pm0.18}$$

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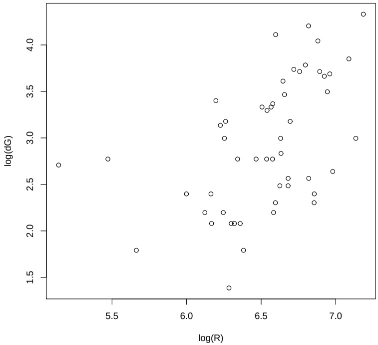

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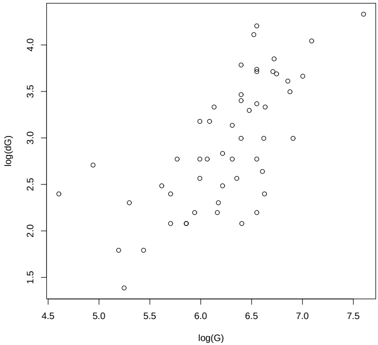

log δG *vs* log G

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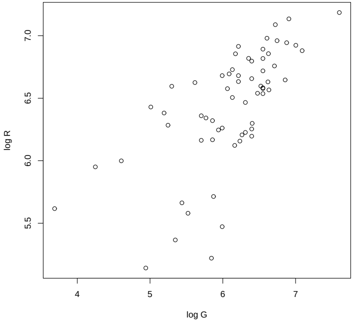
Not so good.

Subtleties

G and R are highly correlated (0.74):



log R *vs* log G

and so the overall powers in the loss-rates, $g_1 + r_2$ and $r_1 + g_2$, are better-determined than their constituents: variation is less significant *along* the lines of constant $g_1 + r_2$ and $r_1 + g_2$ than *orthogonal* to them.

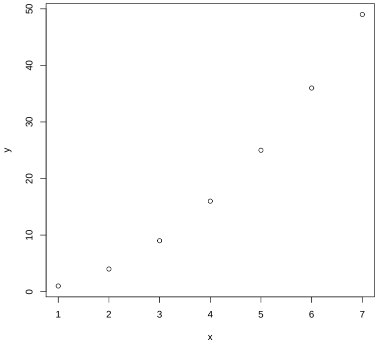
Subtleties

When $g_1 + r_2 \neq 1$ or $r_1 + g_2 \neq 1$, autonomous battles ('raids') should not be aggregated into daily data.

If they are, the effect is to push the overall powers $g_1 + r_2$ and $r_1 + g_2$ away from their true values and towards one, and to reduce the quality of the fit.

Subtleties

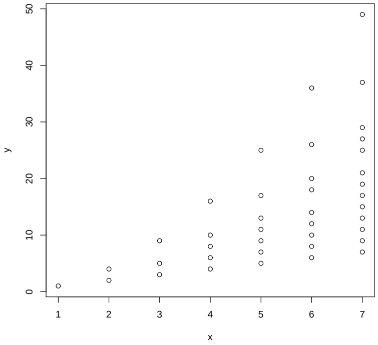
Example: $y = x^2$



has $\log y = 2 \log x$, of course.

Subtleties

Example: $y = x^2$ and sums of these: *e.g.* not only $(3, 9)$ but also $(1 + 2, 1 + 4) = (3, 5)$ and $(1 + 1 + 1, 1 + 1 + 1) = (3, 3)$.



and the best fit is now $\log y = 1.5 \log x$, with $\Sigma R^2 = 0.6$.

Subtleties

Upshot: **asymmetry** is typically **greater** than the data suggest.

The Battle of Britain: Overall

$$\frac{dR}{dt} = -g\,G^{1.12\pm0.17}\,R^{0.18\pm0.25}\,, \qquad \frac{dG}{dt} = -r\,R^{0.00\pm0.25}\,G^{0.86\pm0.18}$$

has $\gamma \equiv 1 + g_1 - g_2 \simeq 1.3$, $\rho \equiv 1 + r_1 - r_2 \simeq 0.8$.

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More accurate are the differences of $g_1 + r_2$ or $r_1 + g_2$ from one:

$$g_1 + r_2 = 1.30, \qquad r_1 + g_2 = 0.86,$$

and thus the **asymmetry**

$$\gamma - \rho = g_1 + r_2 - r_1 - g_2 = 0.44.$$

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We can conclude with fair confidence that $\gamma > 1$ and $\rho < 1$, and with much more confidence that $\gamma > \rho$.

Thus the German attackers may have benefited from mere numbers, all else equal: but the British defenders did not.

Ian Johnson & NJM, Lanchester models and the Battle of Britain, *Naval Research Logistics* **58** (2011) 210-222.

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Rather, to the extent to which $\gamma > \rho$, the RAF had a defender's advantage.

The achievement of Keith Park (Commander, 11 Group, RAF Fighter Command) lay in creating and exploiting this advantage:

'It [is] better to have even one strong squadron of our fighters over the enemy than a wing of three climbing up below them'

NJM & Chris Price, Safety in Numbers: Ideas of concentration in Royal Air Force fighter defence from Lanchester to the Battle of Britain, *History* **96** (2011) 304-325.

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The best engagement-level data we have is for Vietnam.

Vietnam 1965-68; Rolling Thunder

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NVAF (MiG 17,19,21) sorties tend to cause **own** losses, whether against F4s or F105s.

NVAF conclusion: sortie sparingly, disrupt, avoid engagement.

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air combat is approximately linear-law, and **asymmetric**.

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air combat is approximately linear-law, and **asymmetric**.

To the extent to which there is some advantage in numbers, this is true only for the **attacker**. In contrast the **defender**'s optimal tactics are of cover, concealment, dispersal, denial, disruption, force preservation.

Ian Horwood, NJM & Chris Price, Concentration and asymmetry in defensive air combat: from the battle of Britain to the 21st century, submitted to *Air Power Review*.