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Determination of Naval Gun System Firing Patterns to Combat Manoeuvring Surface Targets

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NOTICE

(U) This document has been reviewed and DOES NOT CONTAIN controlled goods.

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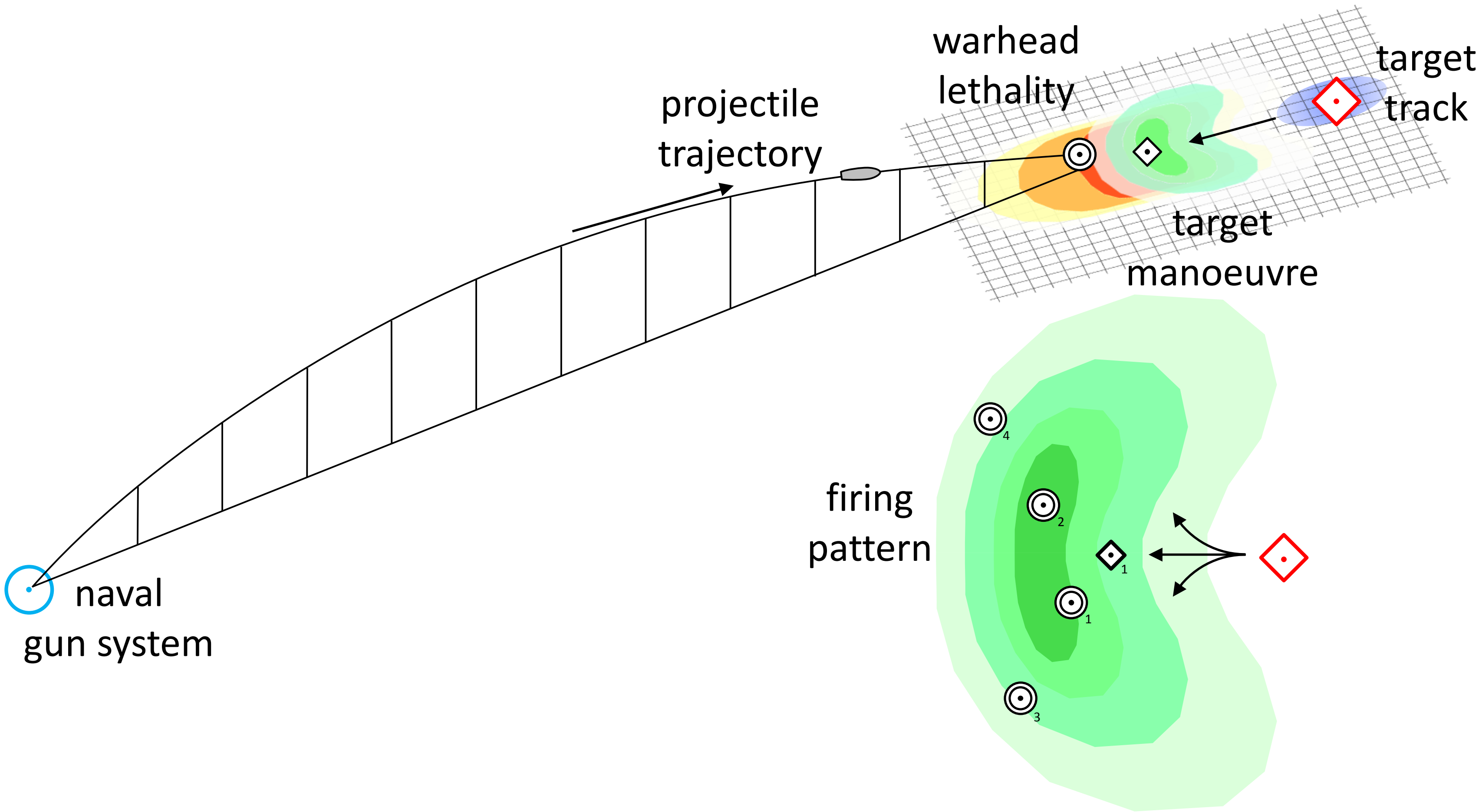
Introduction

- Motivation: Defence of ships from attack by fast manoeuvring boats
- Focus: Employment of naval gun systems against manoeuvring surface targets
- Objective: Determine optimal salvo firing patterns
- Problem characteristics:
  - Target tracking errors (range, bearing, velocity)
  - Target manoeuvre assumptions
  - Warhead size, muzzle velocity, aiming dispersion errors, firing rate
  - Warhead fusing, number of fragments, fragment sizes and initial velocities
  - Target vulnerability
  - Number of rounds in salvo, aim point offsets (height, lead & lateral) from projected target location at time of detonation

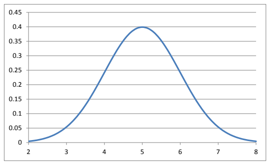
All data and results shown are for illustrative purposes only and do not reflect actual systems.

Problem Overview

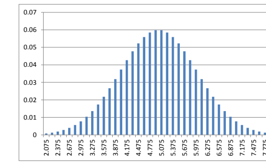


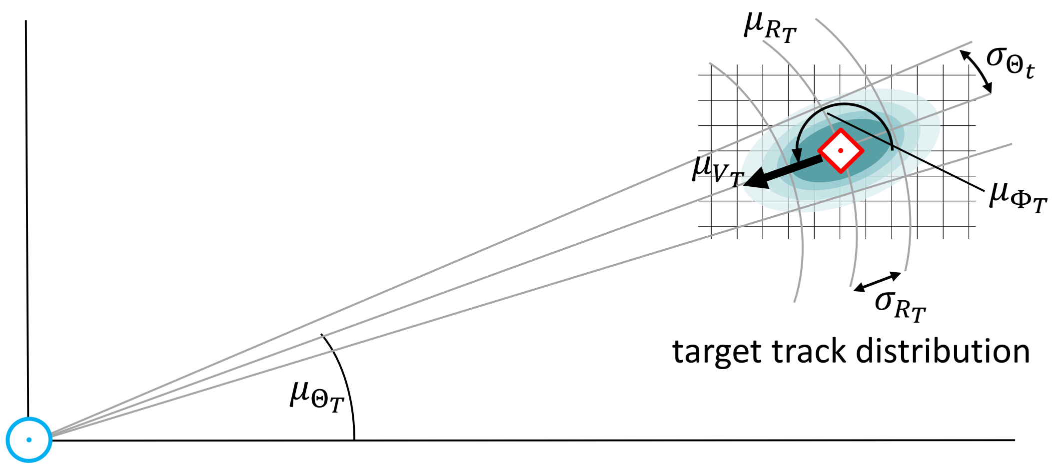
Target Track

- Target tracked by ship Fire Control System (FCS – a radar)
- Tracking errors: Continuous Random Variables approximated as Discrete



range	$R_T = N\left(\mu_{R_T}, {\sigma_{R_T}}^2\right)$	$\overline{R_T} = \{r_{T\_i_r}, P_{R_T\_i_r}\}, i_r = 1 \ldots N_{R_T}$
bearing	$\Theta_T = N\left(\mu_{\Theta_T}, {\sigma_{\Theta_T}}^2\right)$	$\overline{\Theta_T} = \{\theta_{T\_j_\theta}, P_{\Theta_T\_j_\theta}\}, j_\theta = 1 \ldots N_{\Theta_T}$
target velocity	$V_T = N\left(\mu_{V_T}, {\sigma_{V_T}}^2\right)$	$\overline{V_T} = \{v_{T\_k}, P_{V_T\_k}\}, k = 1 \ldots N_{V_T}$
target heading	$\Phi_T = N\left(\mu_{\Phi_T}, {\sigma_{\Phi_T}}^2\right)$	$\overline{\Phi_T} = \{\phi_{T\_l}, P_{\Phi_T\_l}\}, l = 1 \ldots N_{\Phi_T}$





- Target track distribution constructed numerically:

$$P(T_{i,j,k,l}) = \left(\sum_{i_r} \sum_{j_\theta} P_{R_T\_i_r} P_{\Theta_T\_j_\theta}\right) P_{V_T\_k} P_{\Phi_T\_l}$$

Probability target located at grid cell i, j with velocity v_{T_k} and heading ϕ_{T_l}

summation done over i_r, j_θ . ∋. $r_{T\_i_r}, \theta_{T\_j_\theta}$ maps into grid cell i, j

Target Manoeuvre

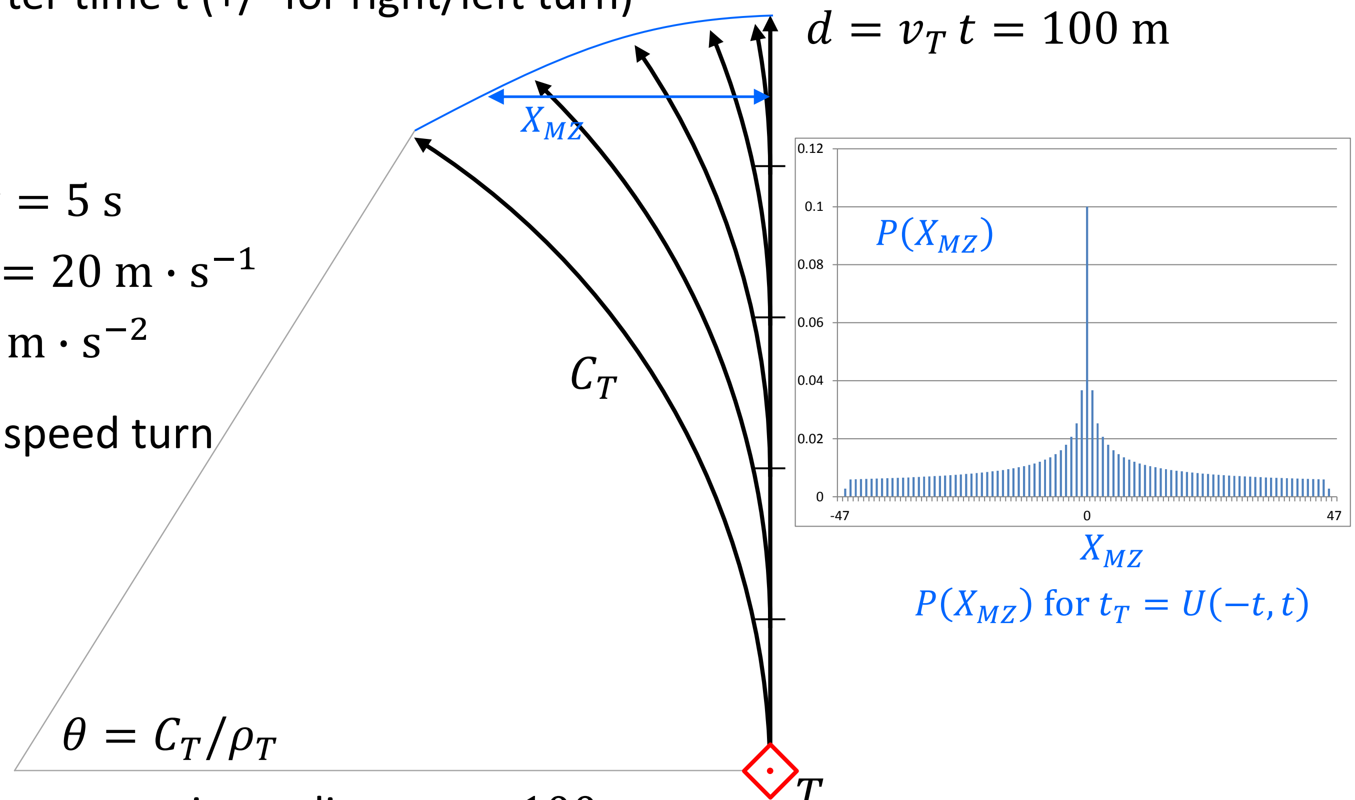
Manoeuvre definition: travel straight t_S seconds followed by turn at maximum rate t_T seconds, where $t = t_S + t_T$ is the fly-out time of the round

Probability distribution over $t_T \in [-t, t]$, e.g. Uniform, gives distribution of target positions after time t (+/- for right/left turn)

Illustrative example:

- projection time: $t = 5 \text{ s}$
- target velocity: $v = 20 \text{ m} \cdot \text{s}^{-1}$
- gravity: $g = 9.81 \text{ m} \cdot \text{s}^{-2}$

$C_T = d$ for constant speed turn

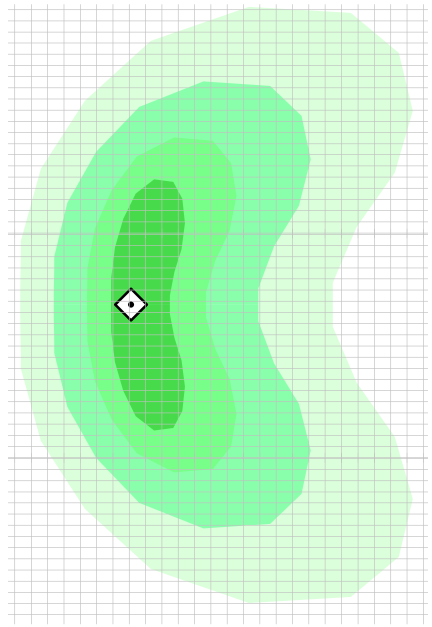


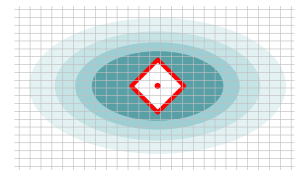
turning radius: $\rho_T = 100 \text{ m}$

turn acceleration: $N_G = \frac{{v_T}^2}{g\,\rho_T} = 0.41 \text{ G}$

Target Movement

Target movement zone
$P\left(L_{i,j} | T_{i,j,k,l}\right)$



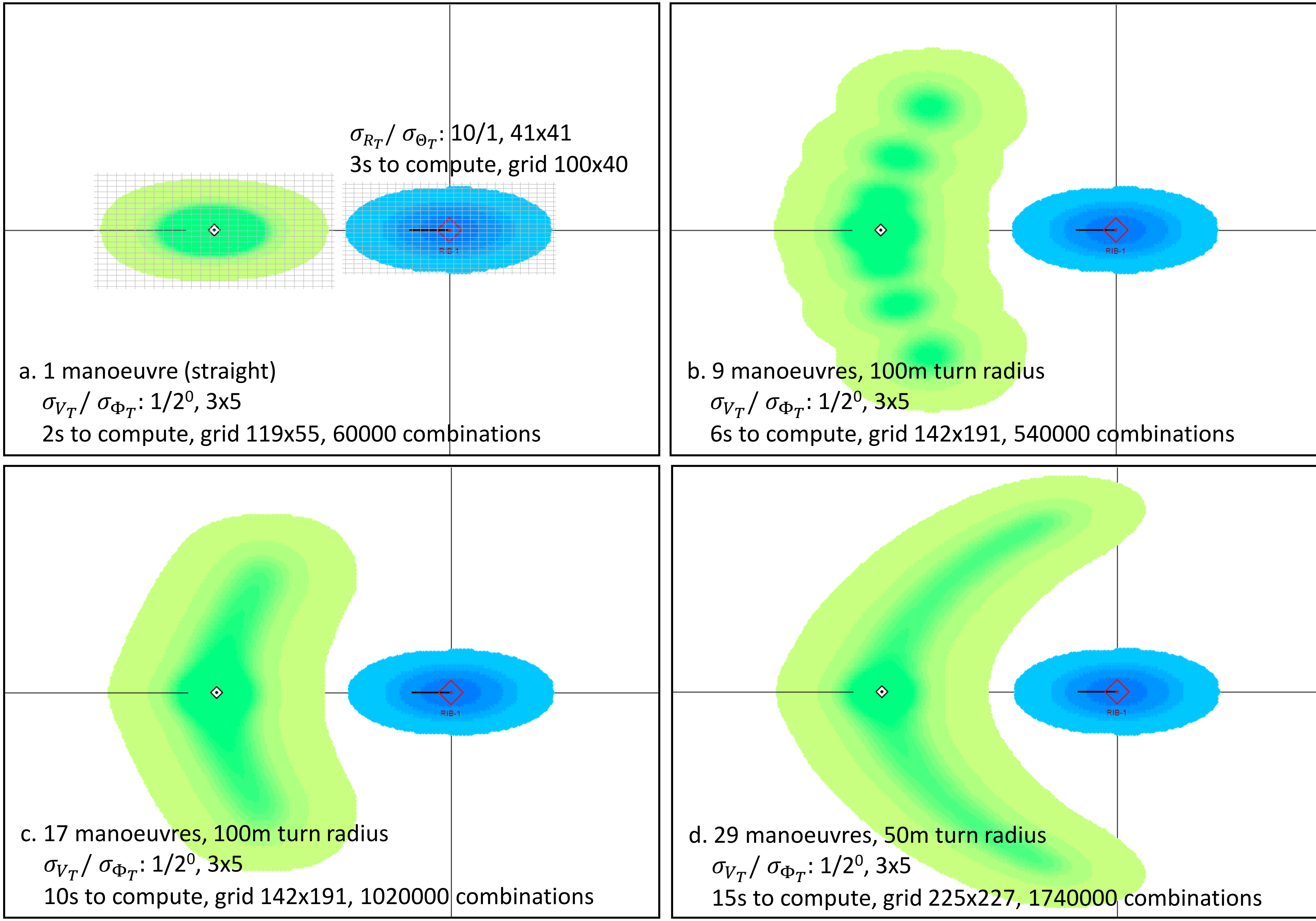


Target track
$P\left(T_{i,j,k,l}\right)$

For each initial target location $T_{i,j,k,l}$ from track distribution

- Apply set of target manoeuvres with corresponding probabilities
- Aggregate final target locations with probabilities to yield the target movement distribution

Example Target Movement Zones



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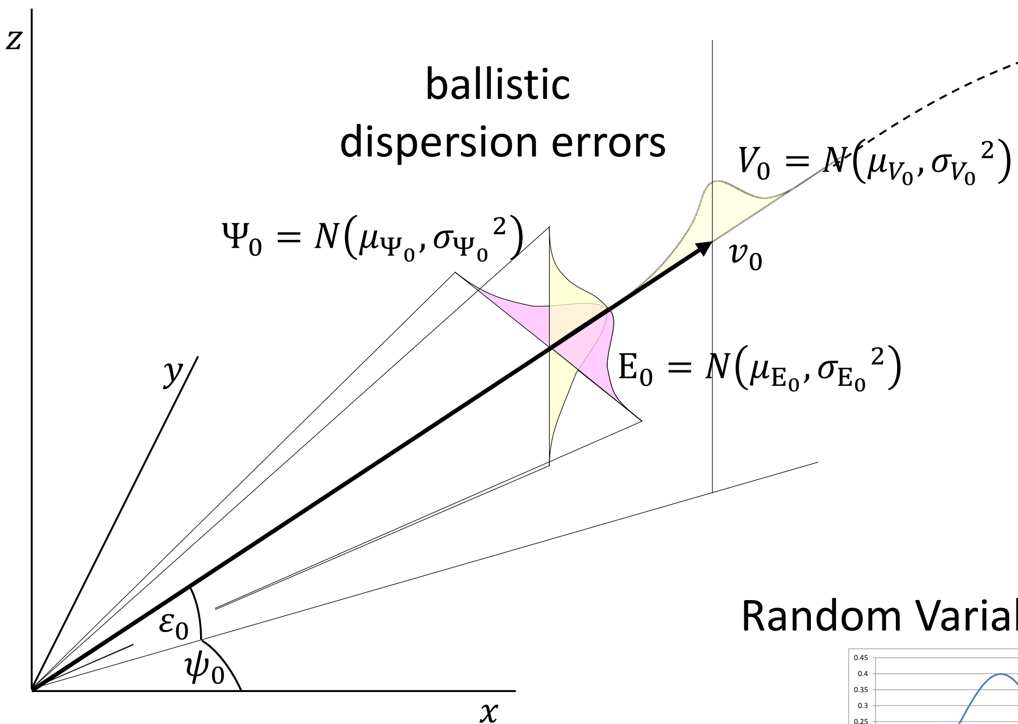
Projectile Trajectory

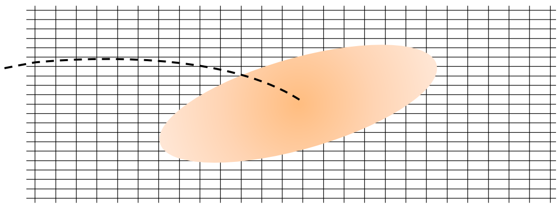
- Initial conditions defining projectile trajectory:
  - muzzle velocity: v_0
  - quadrant elevation angle: ε_0
  - azimuth angle: ψ_0
- Point Mass (PM) equations for motion in the vertical plane solved numerically using forward difference approximation
- Projectile height adjusted at target location for curvature of earth
- Iterative Newton's method used for finding elevation angle required to give burst height / lead offset to target position projected from track position for time of flight
- Model does not account for drift due to rotating projectile and Coriolis effect
- Model gives good accuracy for reproducing results from published range tables
  - range, terminal velocity, angle of descent, time of flight as functions of quadrant elevation within 0.5% for varying ranges

Projectile Dispersion Errors

Muzzle velocity, elevation and azimuth angles represented as normally distributed Random Variables

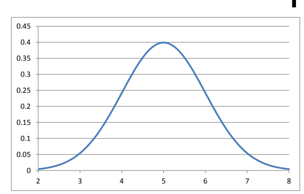
Projectile terminal properties are functions (based on ballistics equations) of Random Variables

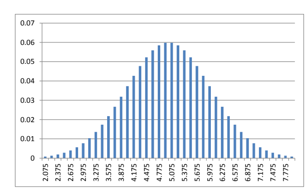




$$X_D = h_1(V_0, \mathrm{E}_0, \Psi_0)$$
$$Y_D = h_2(V_0, \mathrm{E}_0, \Psi_0)$$
$$Z_D = h_3(V_0, \mathrm{E}_0, \Psi_0)$$
$$V_D = h_4(V_0, \mathrm{E}_0, \Psi_0)$$
$$\mathrm{E}_D = h_5(V_0, \mathrm{E}_0, \Psi_0)$$
$$\Psi_D = h_6(V_0, \mathrm{E}_0, \Psi_0)$$

Random Variables approximated as discrete

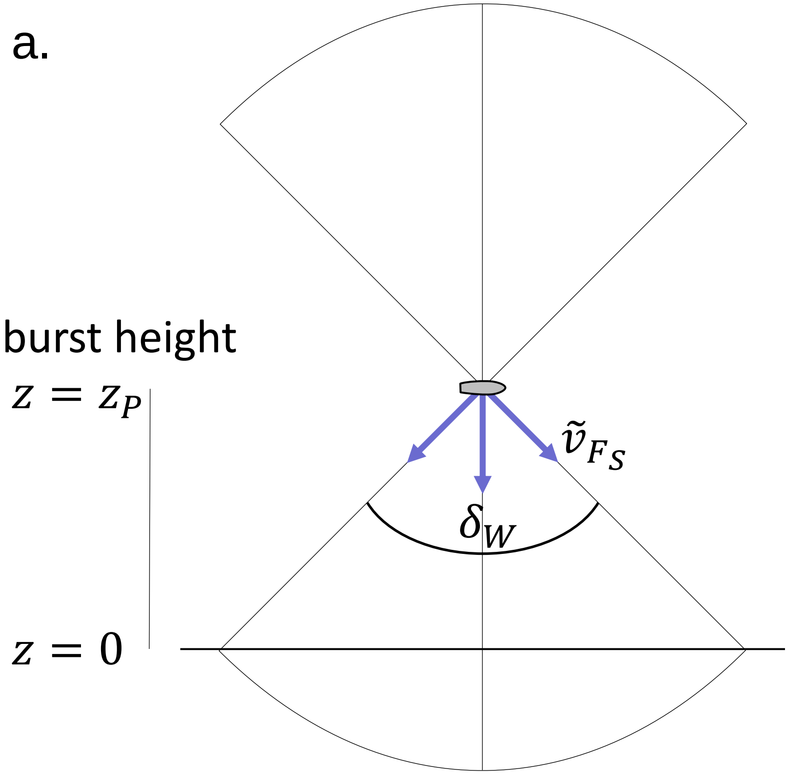




Permits simple (but computationally intensive) determination of detonation point distributions with projectile terminal properties

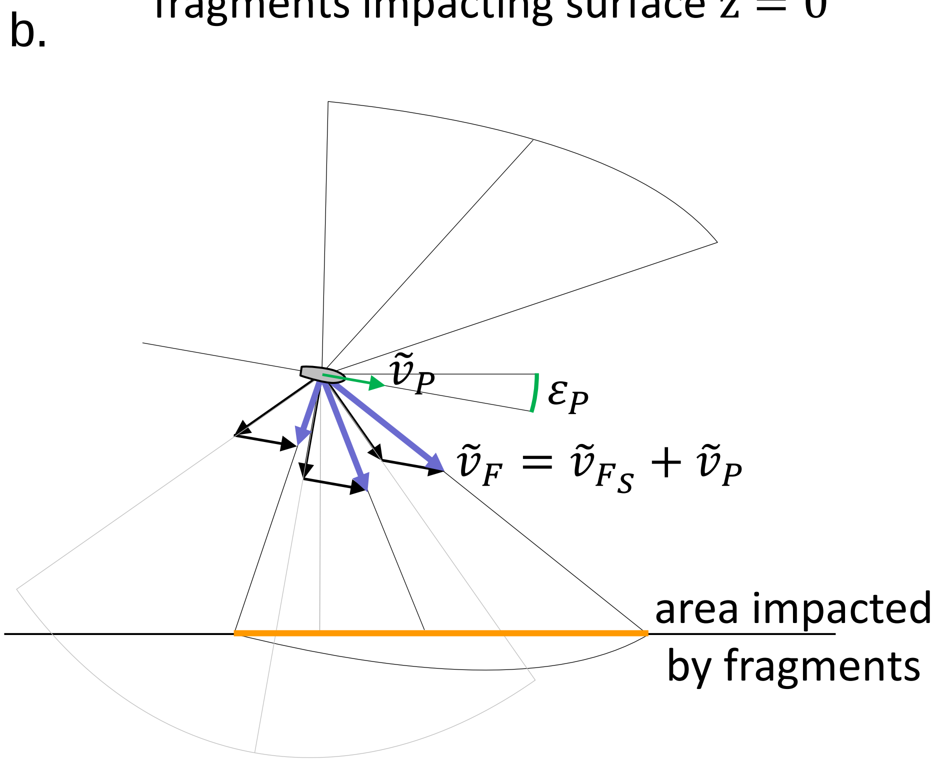
Warhead Fragmentation Model

Warhead detonation: N_W fragments expelled in circular cone ring with initial velocity $\tilde{v}_{f_S}$



Static warhead:
0^0 incidence

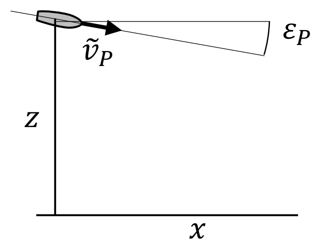
Adjust for moving warhead and determine number and energy of fragments impacting surface $z = 0$



Moving warhead:
ε_P incidence and $\tilde{v}_P$ velocity

Energy of Fragment Impact

$$\tilde{v}_F = \tilde{v}_{F_S} + \tilde{v}_P$$



$$(\tilde{v}_P)_x = |\tilde{v}_P| \cos \varepsilon_P$$

$$(\tilde{v}_P)_y = 0$$

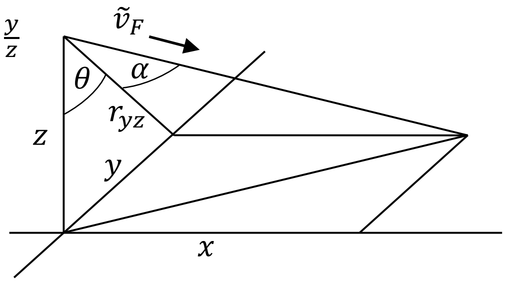
$$(\tilde{v}_P)_z = |\tilde{v}_P| \sin \varepsilon_P$$

$$(\tilde{v}_F)_x = |\tilde{v}_F| \sin \alpha$$

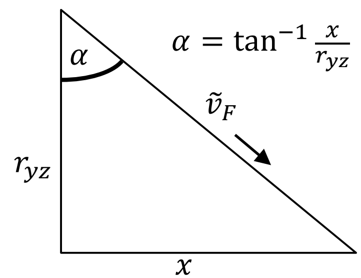
$$(\tilde{v}_F)_y = |\tilde{v}_F| \cos \alpha \sin \theta$$

$$(\tilde{v}_F)_z = |\tilde{v}_F| \cos \alpha \cos \theta$$

$$\theta = \tan^{-1} \frac{y}{z}$$



$$\alpha = \tan^{-1} \frac{x}{r_{yz}}$$



$$\left(\tilde{v}_{F_S}\right)_x = \left|\tilde{v}_{F_S}\right| \sin \bar{\alpha}$$

$$\left(\tilde{v}_{F_S}\right)_y = \left|\tilde{v}_{F_S}\right| \cos \bar{\alpha} \sin \bar{\theta}$$

$$\left(\tilde{v}_{F_S}\right)_z = \left|\tilde{v}_{F_S}\right| \cos \bar{\alpha} \cos \bar{\theta}$$

$$\tilde{v}_{F_S} = \tilde{v}_F - \tilde{v}_P \qquad \left|\tilde{v}_{F_S}\right|^2 = |\tilde{v}_P|^2 + |\tilde{v}_F|^2 - 2|\tilde{v}_P||\tilde{v}_F|\left(\cos \varepsilon_p \sin \alpha + \sin \varepsilon_p \cos \alpha \cos \theta\right)$$

$$|\tilde{v}_F| = |\tilde{v}_P|(\cos \varepsilon_P \sin \alpha + \sin \varepsilon_P \cos \alpha \cos \theta) \pm \sqrt{|\tilde{v}_P|^2(\cos \varepsilon_P \sin \alpha + \sin \varepsilon_P \cos \alpha \cos \theta)^2 + \left|\tilde{v}_{F_S}\right|^2 - |\tilde{v}_P|^2}$$

Fragment ejection velocity in terms of static warhead fragment and projectile velocities

$$v_I = |\tilde{v}_F| - \kappa r_{xyz}$$

Impact velocity at range r_{xyz}
(κ from empirical fit for fragment fly-out)

Spherical model for fragments

$$m_F \frac{d\tilde{v}_F}{dt} = -\tfrac{1}{2} C_D \rho A_F \tilde{v}_F^2$$

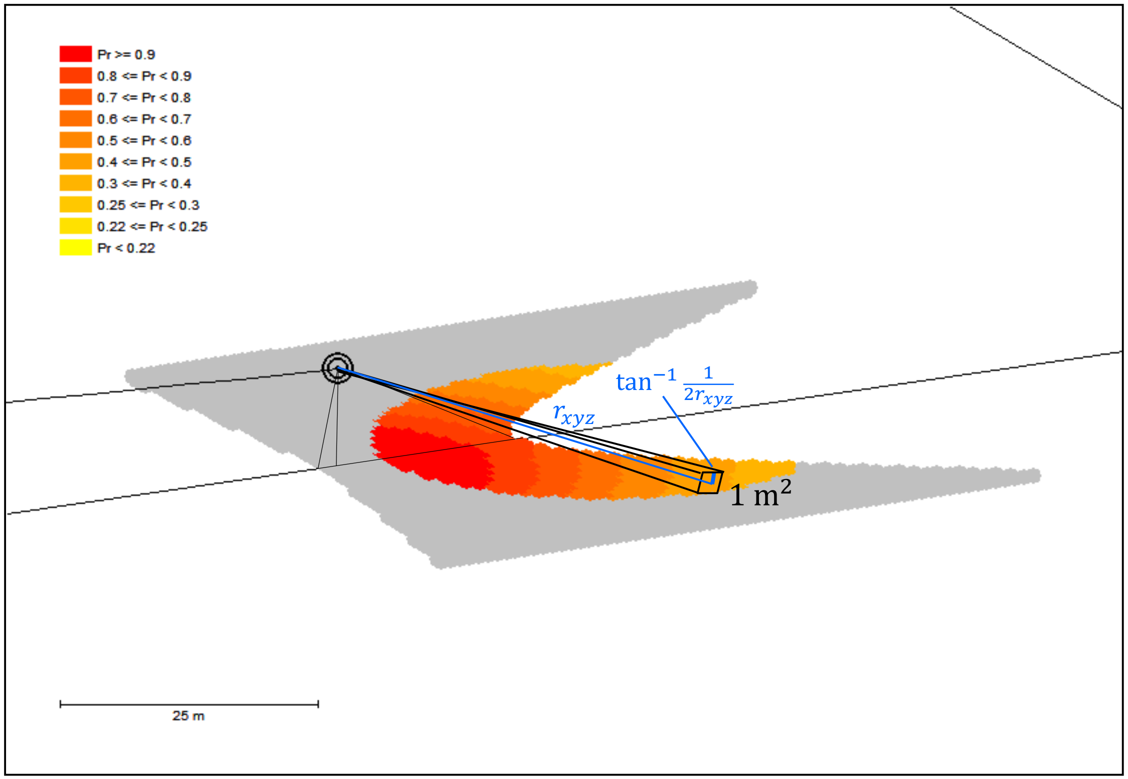
$$\kappa = \frac{d\tilde{v}_F}{dr}$$ approx. const.

$$E = \tfrac{1}{2} m_F {v_I}^2$$

Fragment can cause damage if $E \geq E_T$
(threshold energy, e.g. 80 J)

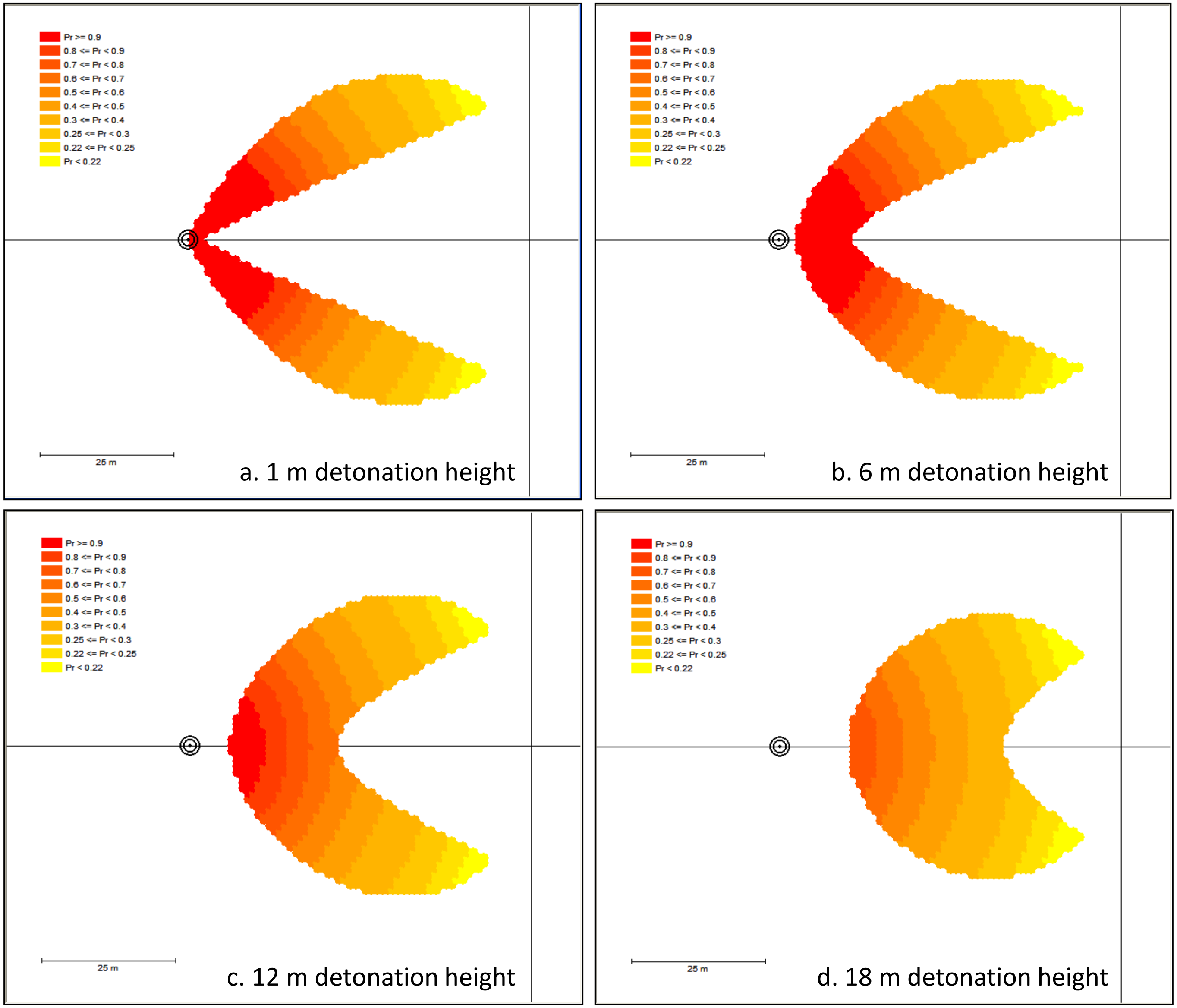
Probability of Incapacitation due to Fragment Impact

$$d_F = \frac{2N_W}{\pi\delta_W}\left(\tan^{-1}\frac{1}{2r_{xyz}}\right)^2 \qquad P\left(K|L_{i,j},D\right) = 1 - e^{-d_F A_T}$$

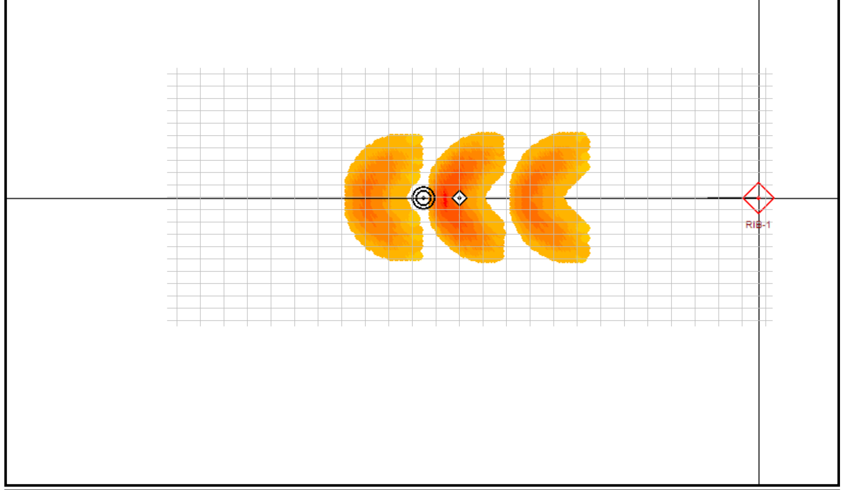


N_W	number of fragments
δ_W	fan angle of fragment dispersion
r_{xyz}	distance to target
d_F	density of efficient fragments per m^2
A_T	vulnerable area of target (m^2)
$L_{i,j}$	target location
D	warhead detonation point

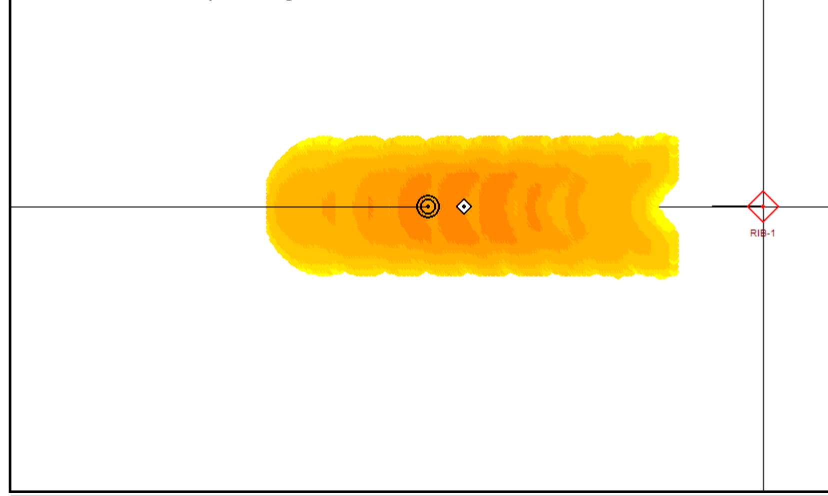
Height Dependency of Fragmentation Patterns



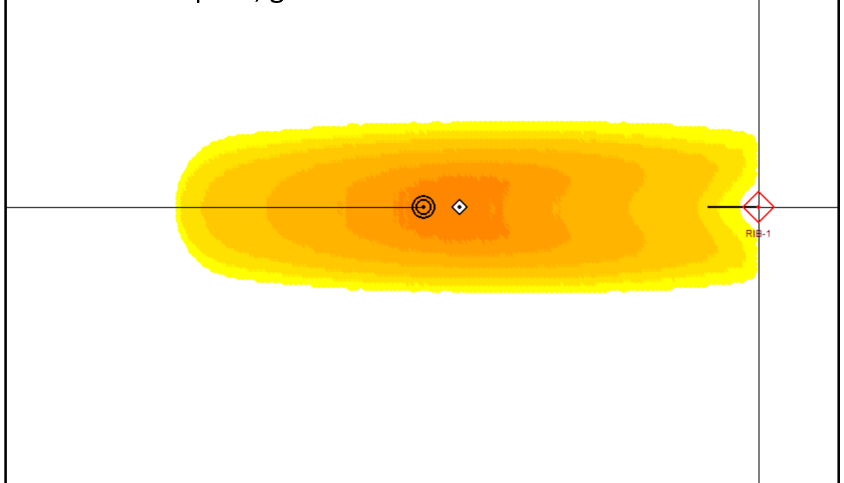
Aim Point Lethal Zone



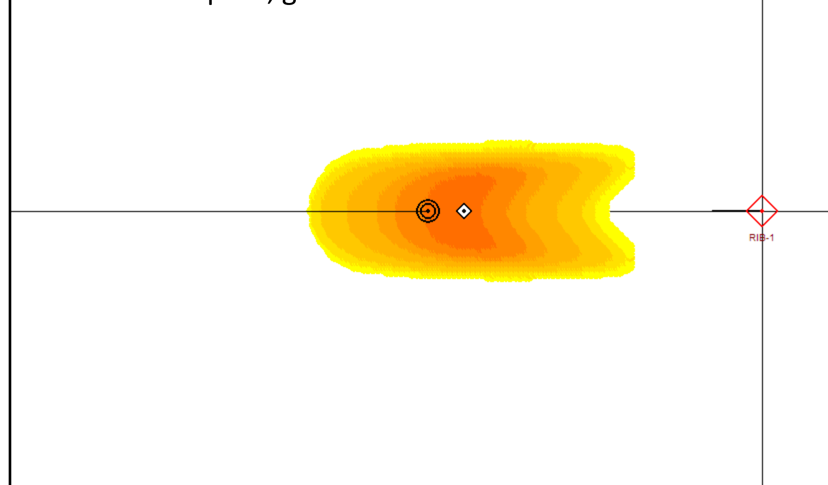
a. σ_{V_0}/ σ_{E_0}/ σ_{Ψ_0}: 10/1/1, 3x3x3, 1x
2s to compute, grid 235x81



b. σ_{V_0}/ σ_{E_0}/ σ_{Ψ_0}: 10/1/1, 9x7x7, 2x
13s to compute, grid 235x81



c. σ_{V_0}/ σ_{E_0}/ σ_{Ψ_0}: 10/1/1, 25x11x11, 3x
78s to compute, grid 235x81



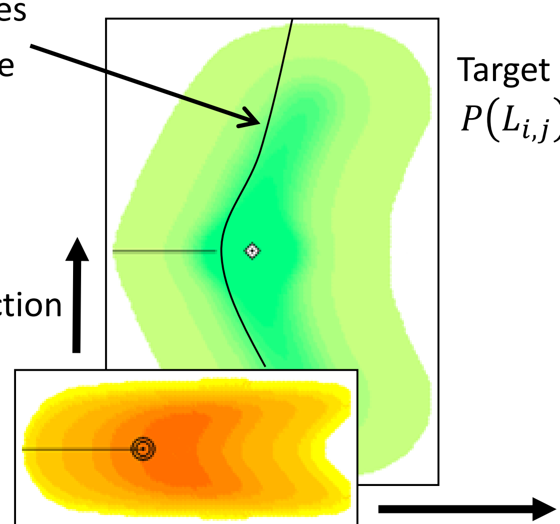
d. σ_{V_0}/ σ_{E_0}/ σ_{Ψ_0}: 5/0.5/0.5, 25x11x11, 3x
51s to compute, grid 135x61

Probability of Kill Assessment

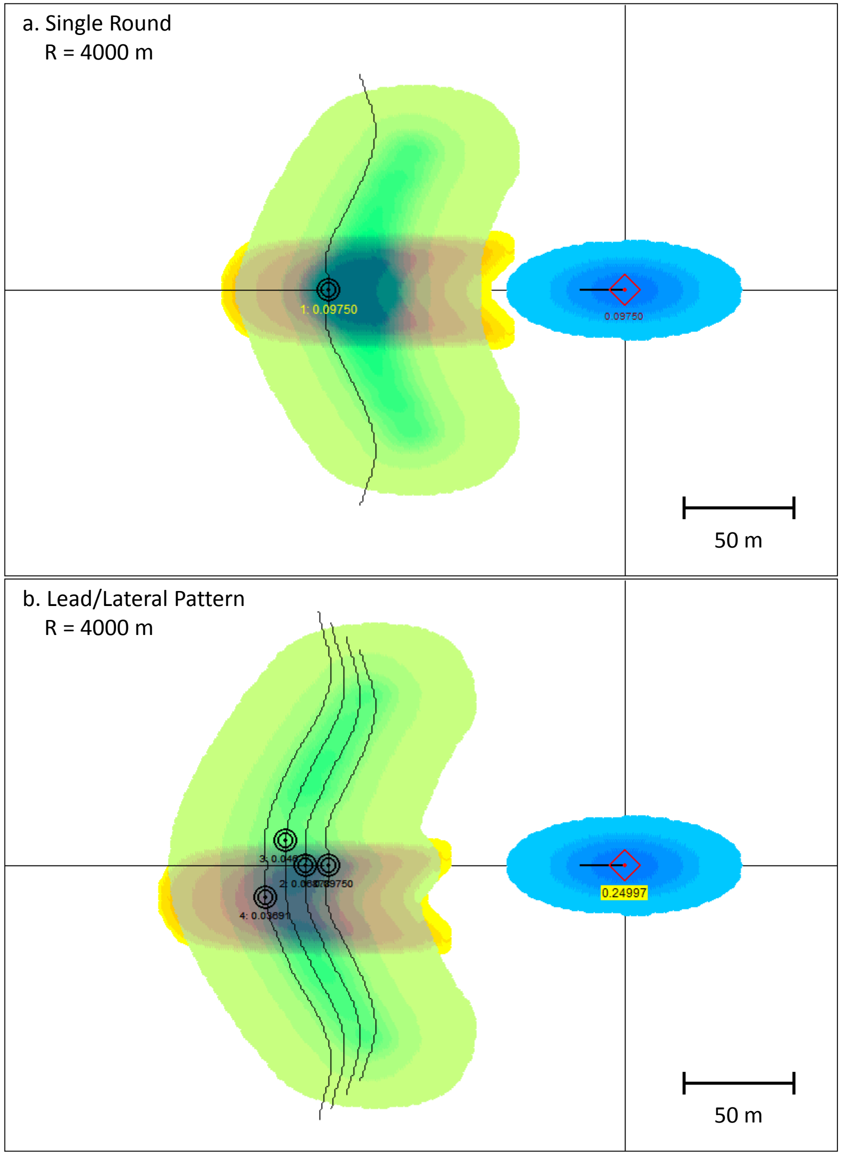
Vary aim point location across all possible points (i_{APt}, j_{APt}) where the aim point lethal zone overlaps the target movement zone

- For each j_{APt}
  - For each i_{APt}, compute $$P(K|A_{i_{APt},j_{APt}}) = \sum_i \sum_j P(L_{i,j}) P(K|L_{i,j}, A_{x_{APt},y_{APt}})$$
  - Obtain the maximum P(K) for the current j_{APt} $$P(K)_{max_{j_{APt}}} = \max_{i_{APt}} P(K|A_{i_{APt},j_{APt}})$$
- Take the maximum across all j_{APt} $$P(K)_{max} = \max_{j_{APt}} P(K)_{max_{j_{APt}}}$$

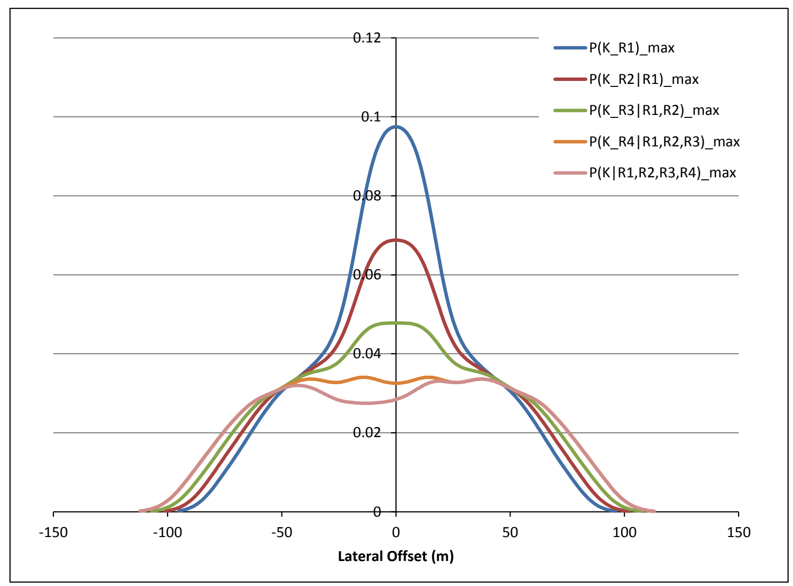
Optimal aim point defined by (i_{APt}, j_{APt}) that gives $P(K)_{max}$



Single Round / Four Round Firing Pattern Example

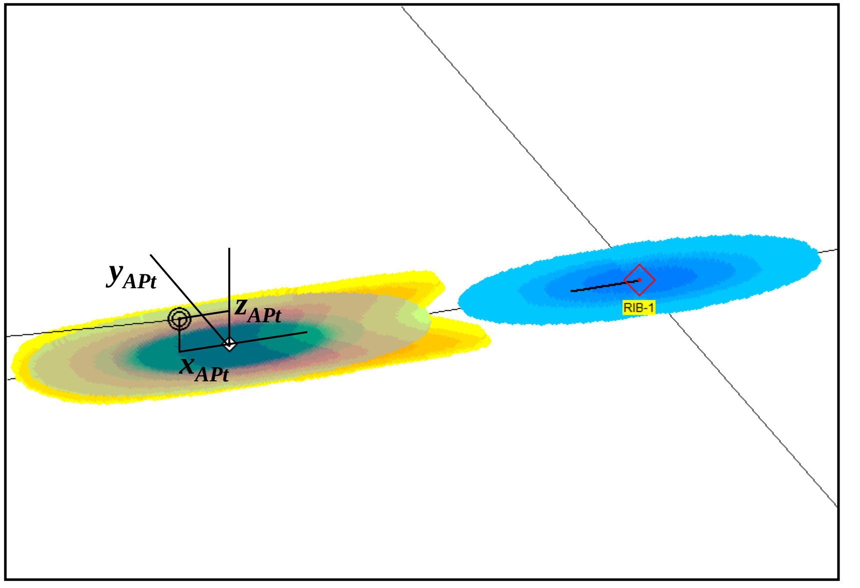


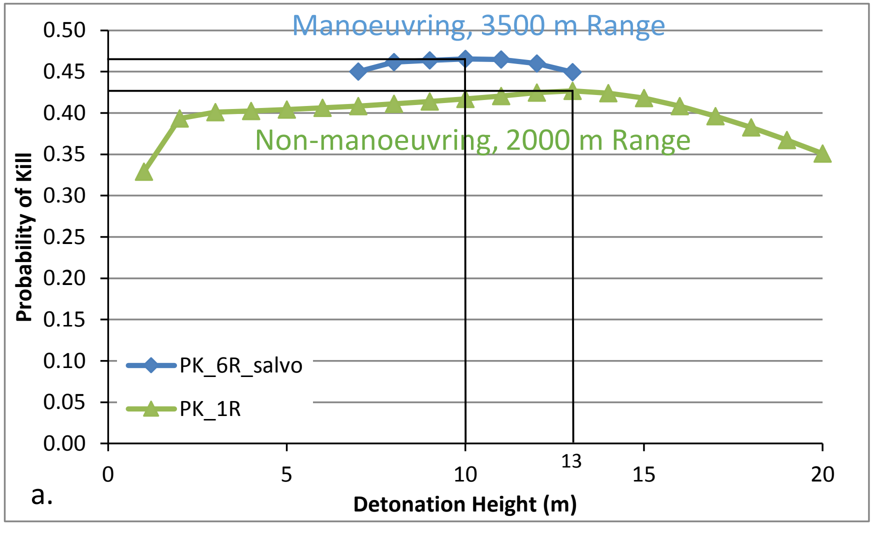
- Single Round P(K) = 0.09750
- Four Round P(K) = 0.24997
- Movement zone shows translation and expansion from first to fourth rounds
- Max P(K) curve for first round shows characteristic spike for 0 lateral offset, reflecting manoeuvre model assumptions
- Further P(K) curves show broadening and reduction due to effects of previous rounds

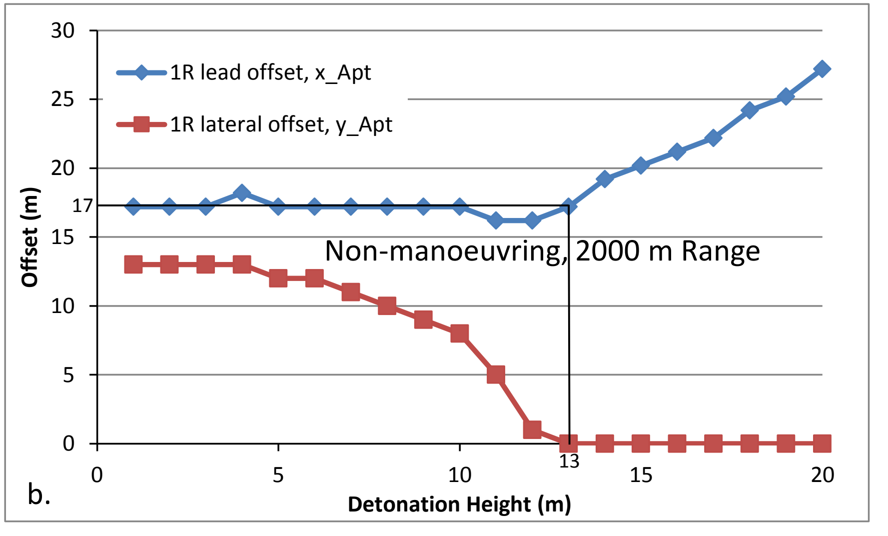


Determination of Optimal Aim Point Offset

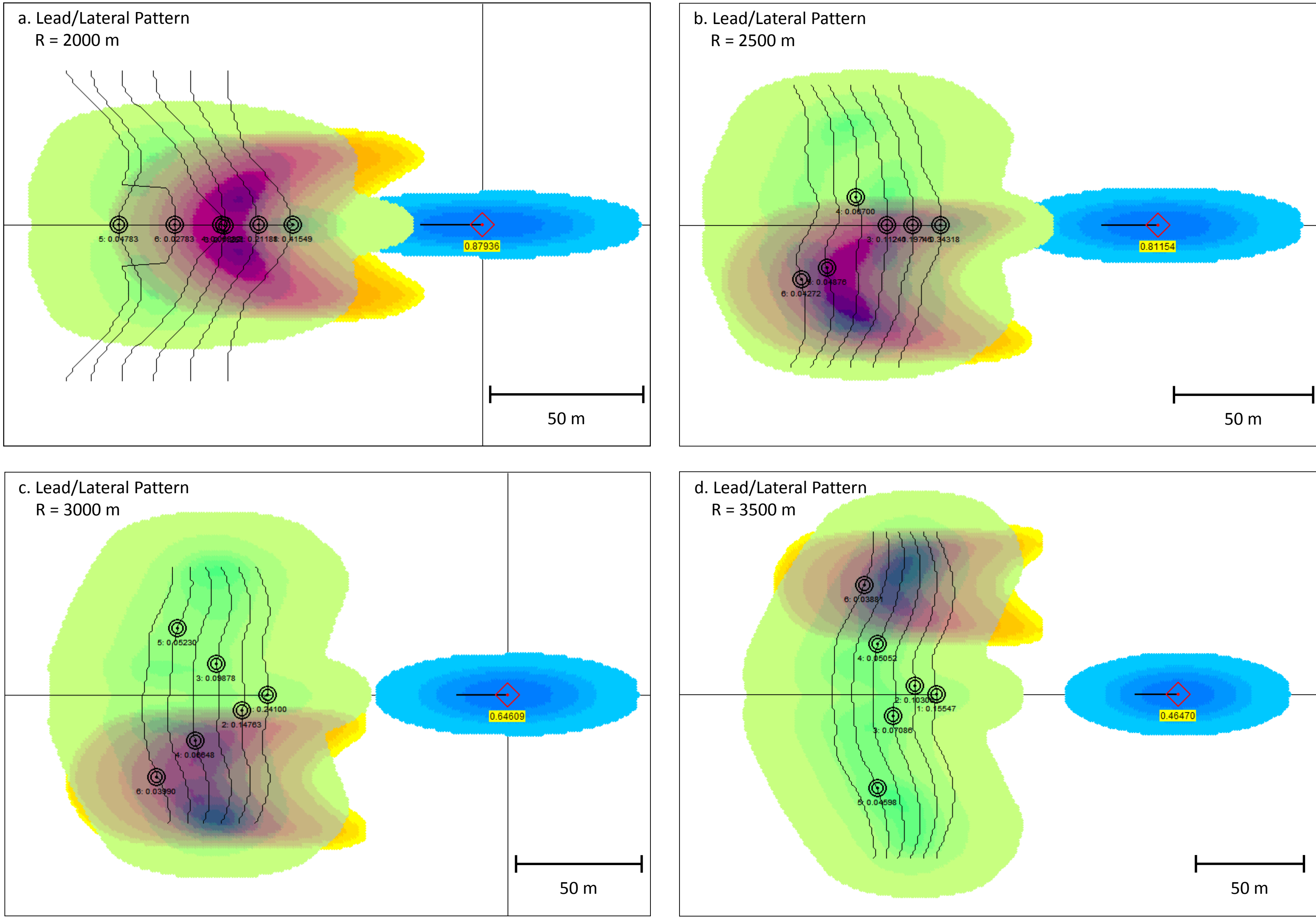
- For given range, aim point burst height varied from 0 to 20 m
  - For each height, aim point lethal zone generated, then used to determine maximum P(K) curve given target movement zone
  - x_APt and y_APt offsets determined from maximum of p(K)_max curve



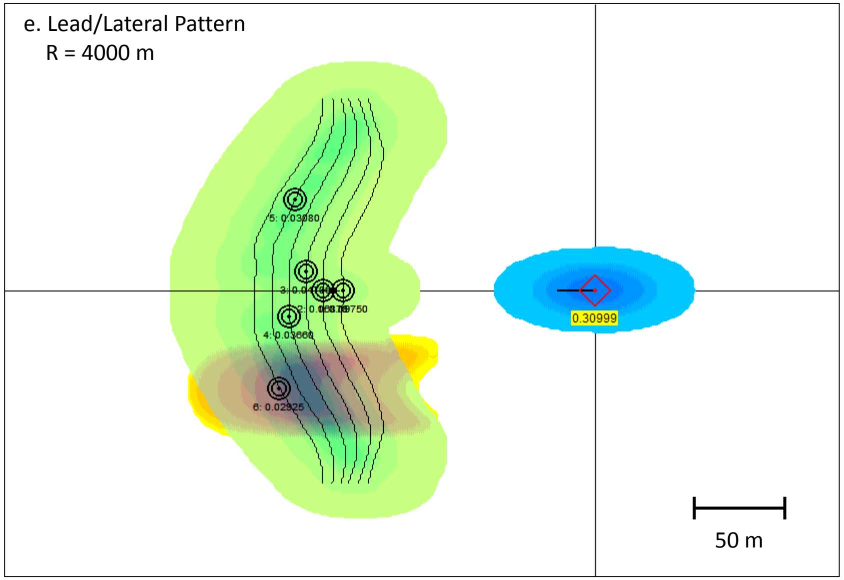


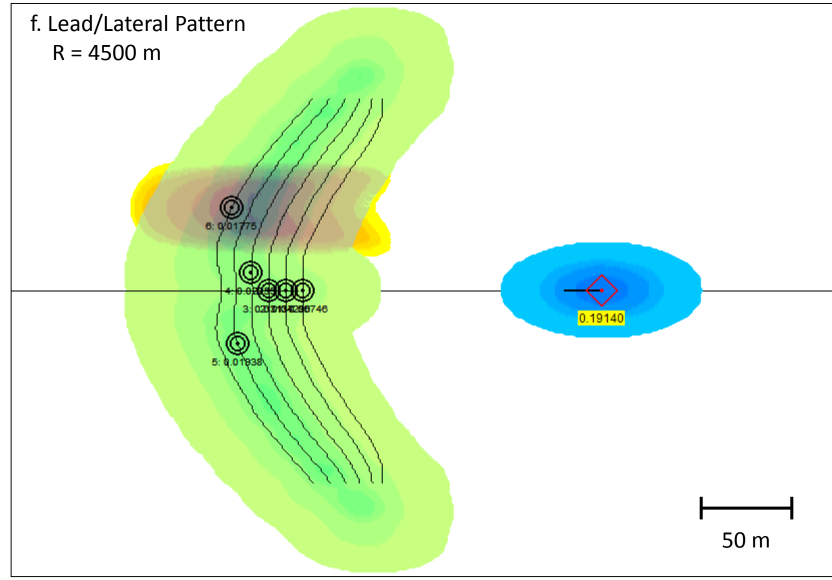


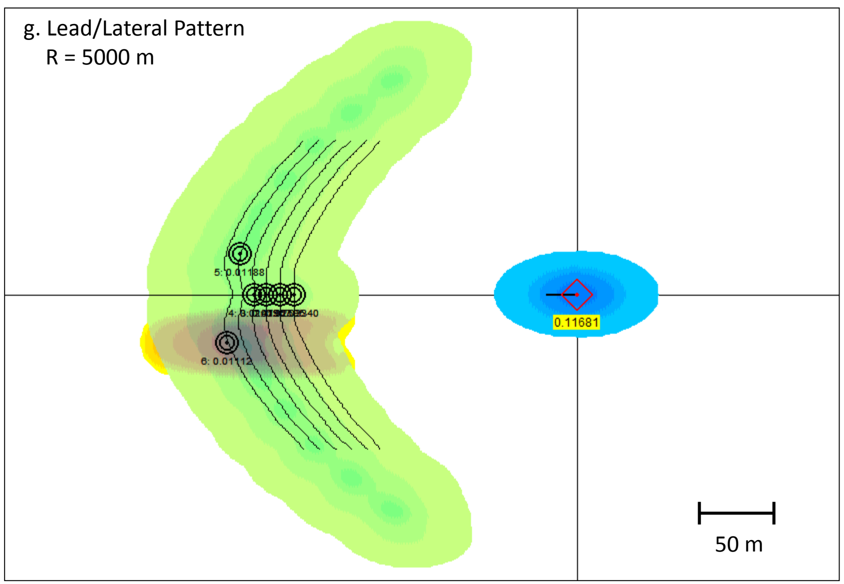
Six Round Firing Pattern Examples (1 of 2)

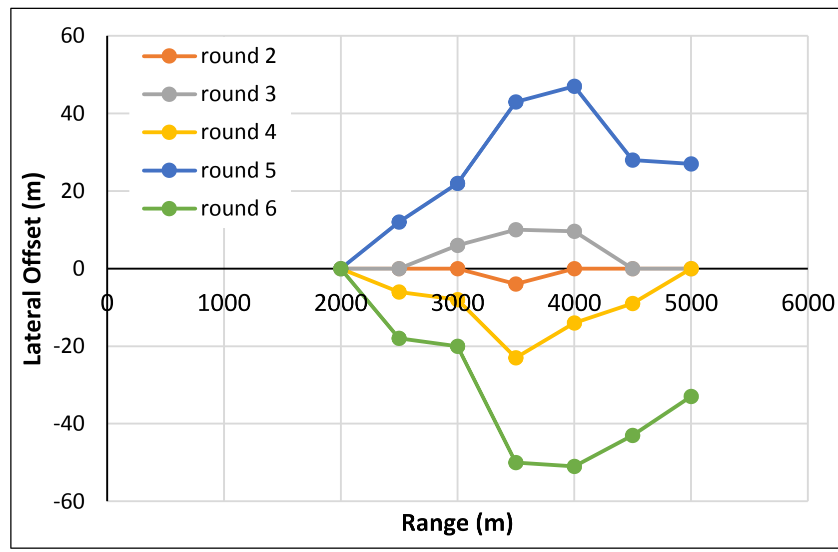


Six Round Firing Pattern Examples (2 of 2)

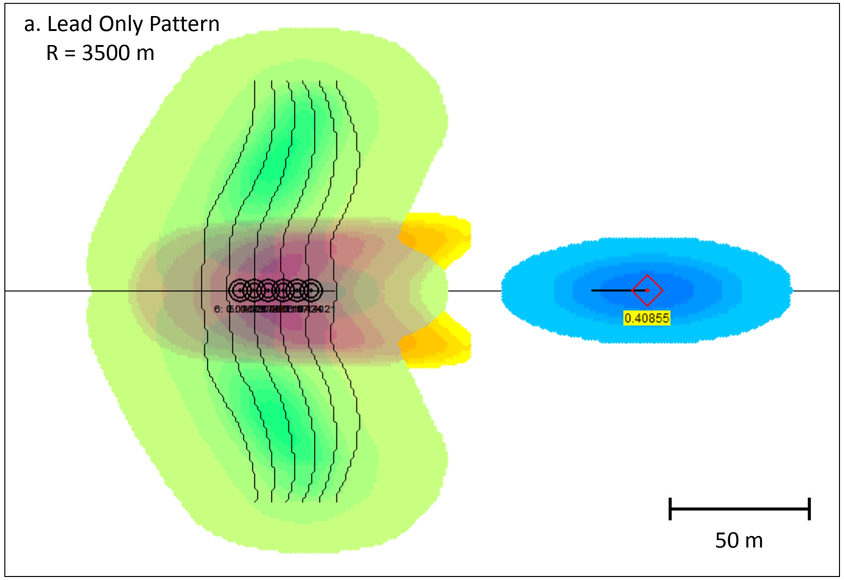


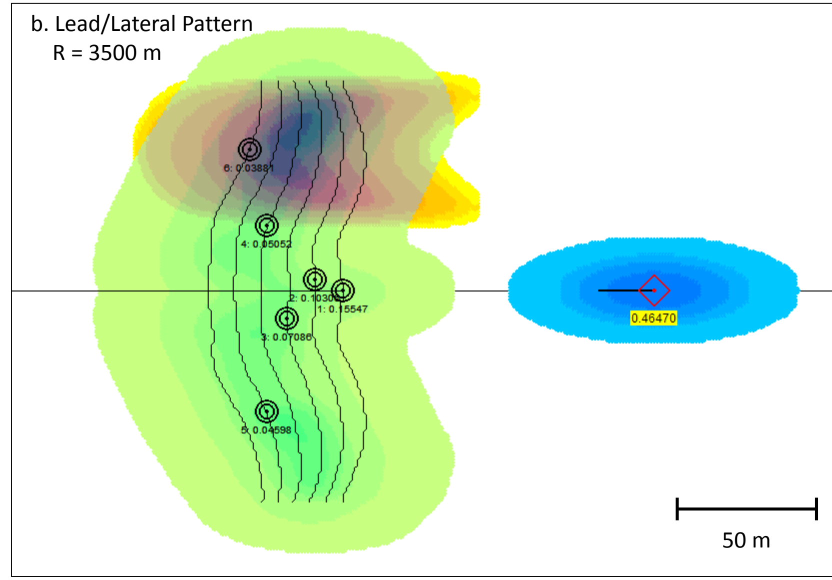


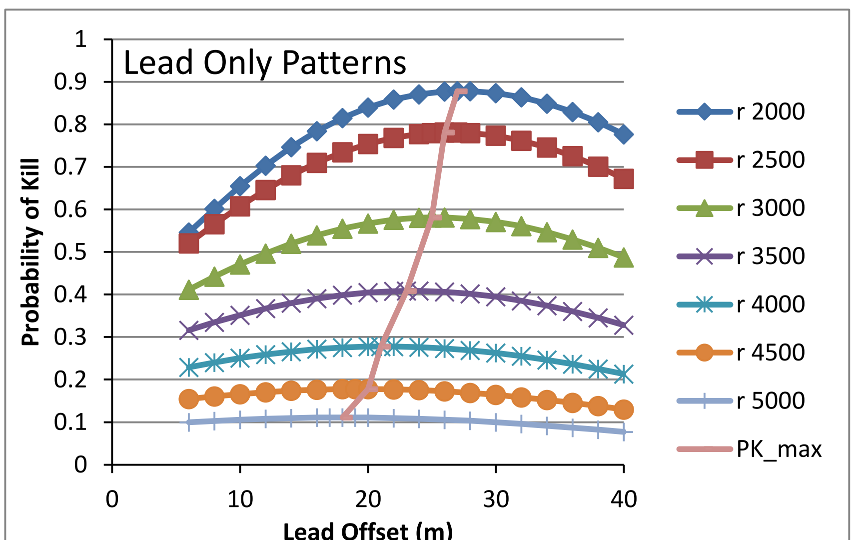


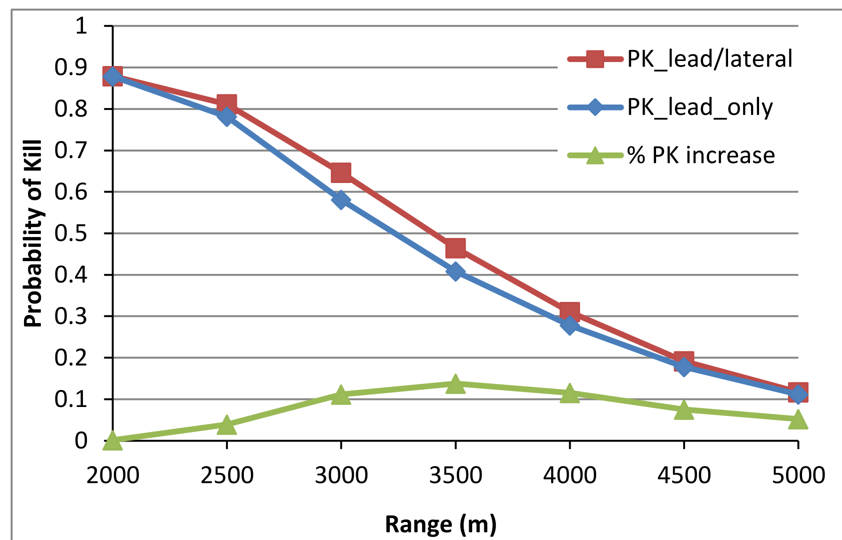


Six Round Lead Only vs. Lead/Lateral Patterns

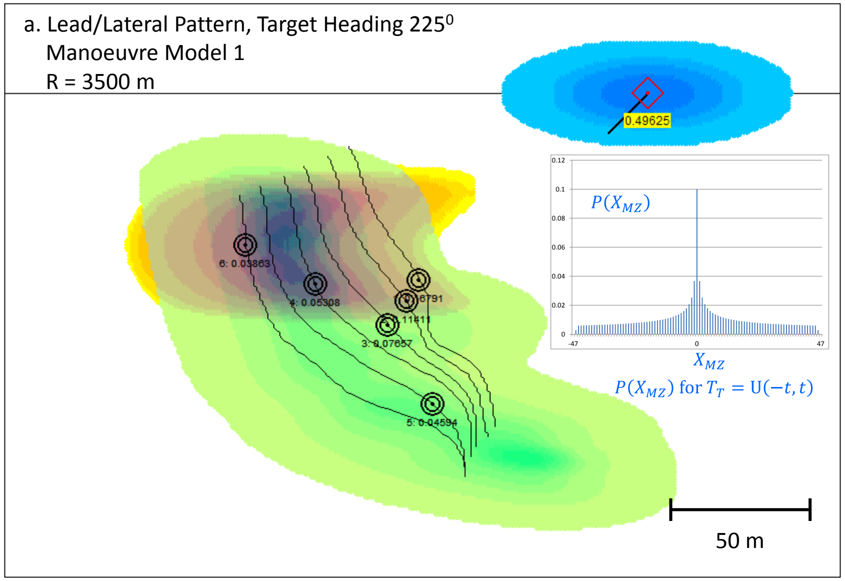


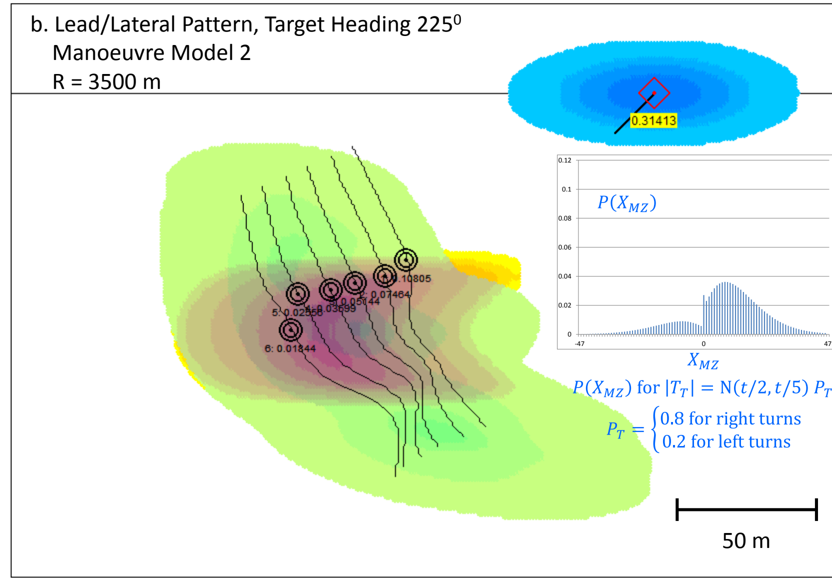






Alternative Target Manoeuvre Assumptions



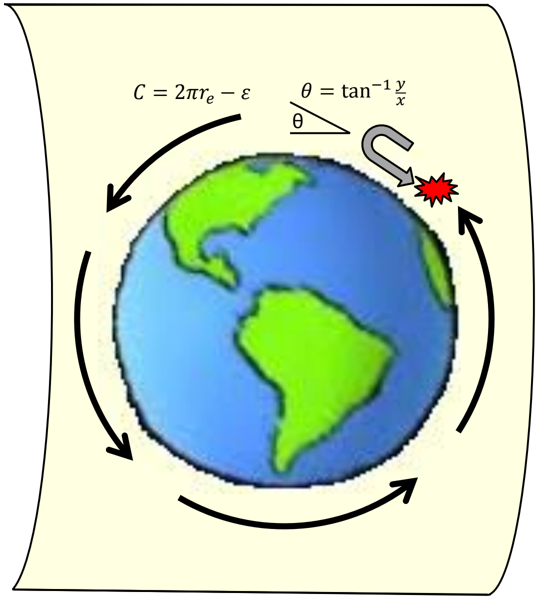


- Target heading turned 45^0 to left, previous manoeuvre model used
  - Target movement zone compacted and skewed with rotation and stretching
  - Lead/lateral firing pattern has increased PK = 0.49625 (v. 0.46470 for previous slide)
- Alternative manoeuvre model used
  - Time turn initiated focused at midpoint of fly-out interval (possible target reaction to firing flash)
  - 1:4 turning bias to right (towards the ship)
  - Lead/lateral firing pattern concentrated on right with decreased PK = 0.31413

Conclusions

- Computational methods developed for determining salvo P(K) from warhead/aim point lethal zones and target movement zones
  - Ballistic model (validated using available range tables)
  - Warhead fragmentation model (simplistic but yields representative behaviours)
  - Target movement zone
- Further work
  - Modified Point Mass (MPM) ballistic model introduced to account for drift
  - Refine warhead fragmentation model and data inputs for warheads of interest
  - Incorporate track updates when calculating target movement zones
  - Consider more sophisticated target manoeuvres
  - Investigate further techniques for finding optimal firing patterns using warhead lethal zones integrated over target movement zones
- Work supports ongoing CFMWC efforts in this area using a mix of modeling (Monte Carlo) and field trials

Questions ?



"and the boffins say, this part of the pattern is in case the target makes a U-turn!"

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