



34 ISMOR workshop: How do we improve Lanchester combat models for defence decision-making?

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Introduction

There is a considerable body of evidence that modern ground combat, a great majority of which is still dismounted combat, follows a pattern that is much closer to Lanchester's linear law than his square law. Assuming this is the case, it raises many harder questions regarding what we do about it. What do we understand are the mechanisms by which this happens? How do we implement these in combat models? How do we gather and prepare the new lethality data? This also goes to the heart of the quality-quantity debate that is as old as operational research (OR), and qualitatively, much older. What are the wider implications?

These questions formed the basis of a workshop held on 19th July 2017 at the 34th International Symposium on Military Operational Research (34 ISMOR), at Royal Holloway College, Egham, in Surrey. There were 11 attendees including the author, of whom approximately half were experienced land battle modellers. This paper records the discussion. It was designed to follow from the present author paper given to the 34 ISMOR plenary the previous day.

The aim of both the paper and the workshop was to increase awareness of how historical analysis (HA) can be used to validate and improve constructive combat models, in order to provide better quality advice on policy and operational decisions. The slides presented at the workshop are included at the end of the paper.

1. Statement of Lanchester's laws of combat

The workshop started with a brief recap of Lanchester's laws of combat. The best known is Lanchester's **square law** for individual direct fire (DF), which states that:

$$dB/dt = -kb.R, \quad dR/dt = -kr.B$$

The most important consequence is that a force's effectiveness is the product of individual unit effectiveness, and the *square* of the numbers in the force. This means that numbers dominate quality in any military conflict that obeys this law.

Lanchester also formulated a **linear law** for area fire, for example by artillery, and which he also applied to ancient battles, characterising them as a series of duels:

$$dB/dt = -kb.R.B, \quad dR/dt = -kr.B.R$$

For battles that follow the linear law, force effectiveness is the product of individual effectiveness and the numbers in the force. This means that quality and numbers are equally important. As presented by Syms (2017), the weight of historical evidence favours using the linear law for most modern land combat.

There have been many extensions and developments to these laws. Smith (1965) proposed a **mixed law**, in which the attrition of each side was represented by a sum of linear and square law components.

Peterson (1967) proposed a **logarithmic law**, in which the attrition that was experienced by a side depended only on the numbers in that side, regardless of enemy numbers:

$$dB/dt = -kb.B, \quad dR/dt = -kr.R$$

This models the not inconsiderable loss rates that every army experiences from disease and non-battle injuries (DNBIs), and those from friendly fire.

Finally, Bracken (1995) proposed a **general law** in which,

$$dB/dt = -kb.R^p.B^q, \quad dR/dt = -kr.B^p.R^q$$

Where p and q are constants, and not necessarily integers. If they are 1 or 0 of course it is possible to derive any of the above laws, hence the generality; in practice, fitting historical data from the Ardennes in 1944 led to values closer to 0.5, indicating that real combat lay somewhere in between the two extremes. When Lucas and Dinges (2004) attempted to fit daily attrition rates from the battle of Kursk in 1943 to Bracken's general law, they also found that different phases of the battle fitted different values of p and q .

Note that in practice, given the large amount of variance in any HA study, it is almost impossible to distinguish between a mixed law that contains summed elements of the square, linear and logarithmic laws, and a fit using fractional powers in the general law.

2. How does this work in reality?

The following possibilities were discussed by the workshop. The present author has taken the opportunity to expand some of these points, noting (where possible) the sources that were quoted, explicitly or by implication.

Anti-personnel fire: The workshop agreed with the point made by Lepingwell (1987), which was confirmed by Rowland (1987) using HA, that anti-personnel fire acts in a Lanchester linear fashion. Not only does this apply to artillery and mortars, which were the biggest cause of casualties in twentieth century combat, but also to anti-personnel DF. Because dismounted personnel form the great majority of the vulnerable population – even at Kursk, 98% of soldiers walked into battle and fought in the open – the overall nature of ground combat is dominated by dismounted combat.

It was observed by Syms (2017) that it was easy to build a Lanchester square model without explicitly doing so, or even being aware that one had done so. If, in a direct fire model, a shot that is aimed at a target evaporates if it fails to hit that specific target, then the model *will* behave in a Lanchester square law fashion. This applied to the US/UK Janus wargame, and combat simulations such as Carmonette, CASTFOREM, WEBS and ATLAS. While this is justified in the case of anti-armour engagements (see the validation by Pizer, 1984), since a shot that is aimed at one enemy tank is very unlikely to hit another, the same cannot be said of

a burst of machine-gun fire aimed at a group of personnel. If the target array is doubled in density, then the casualties will also be doubled.¹

One participant suggested that close assaults onto the objective, if achieved, may follow the square law. It was thought that evidence from past UK work on close combat supports this.

Suppression and fatigue were both noted to reduce the rates that both the attackers and the defenders could engage, slowing down the battle as a whole, but there was no consensus as to how these changed the nature of the battle in relative Lanchester terms.

Terrain effects: The terrain determines the numbers of approach routes that are available to an attacking force, and thus the number that need to be covered by the defender. This in turn influences the force density that is required to achieve a given objective, be this attack or defence. Line-of-sight (LoS) opportunities also affect the optimum balance between direct and indirect weapons, since the former depend on LoS to a greater extent than the latter.

It was noted that while the terrain effectively divides a battle into a number of few-on-few sub-battles, when summed, the Lanchester outcome is the same (though it could change the spread of outcomes). Urban warfare is an extreme example of this compartmentalisation.

The presence of **microterrain** is important in slowing the attacker's movement, allowing longer engagement opportunities for the defender, and greatly reducing the vulnerability of suppressed personnel. One contributor reminded us of the findings of a World War 2 (WW2) British Army Operational Research Group study of artillery suppression.

Engagement frontage effects: The present author raised the issue of frontage. Sabin (2007) noted that the frontage of an ancient army was proportional to the square root of the numbers of personnel in that army, and the present author discovered that attack frontages quoted by Isby (1988) for Soviet Russian doctrine followed exactly the same rule.

Consider one case in which a force attacks with 1,000 personnel on a 1km frontage, and the numbers of defenders engaged will be those who are covering that 1km of front, say 300. If the attacker decides to double the number of attackers to 2,000, then the frontage will increase to 1414m, and the number of defenders available to contribute to the fight will increase to 424, causing the attack casualties will rise proportionally.

ISTAR effects: Surveillance and target acquisition, and how well troops are trained to take advantage of its output, determine the engagement opportunities that are available. Explicitly, if a force is in a target-poor environment, then increasing the numbers of enemy will increase the opportunities that an individual will have any target to engage. This is clearly a Lanchester linear effect. Only when a force is saturated with potential targets will adding another target to the array not result in more being engaged.

A participant made the point that in almost all modern combat, ISTAR influences who fires first, and this in turn creates a huge advantage in that particular 'micro-battle'.

Effects of scale: Much of the discussion focussed on the effects of scale, in that the nature and dimensions of combat may shift from being closer to the square law at low tactical levels, tending towards the linear law at higher (i.e. formation) levels. One participant believed that this was a mathematically inevitable, but the present author will not attempt to explain the proof! This would explain the tendency of HA validation studies to conclude that combat was closest to

¹ The same applied to the massed rifle fire from the late nineteenth century and into WW1, which acted in many ways like a collective machine-gun.

a Lanchester linear phenomenon, since the bulk of data points that have contributed to these analyses (e.g. see McQuie, 1988) are at the corps and divisional levels. The potential shape of a generalised law across different organisational levels is shown in Figure 1.

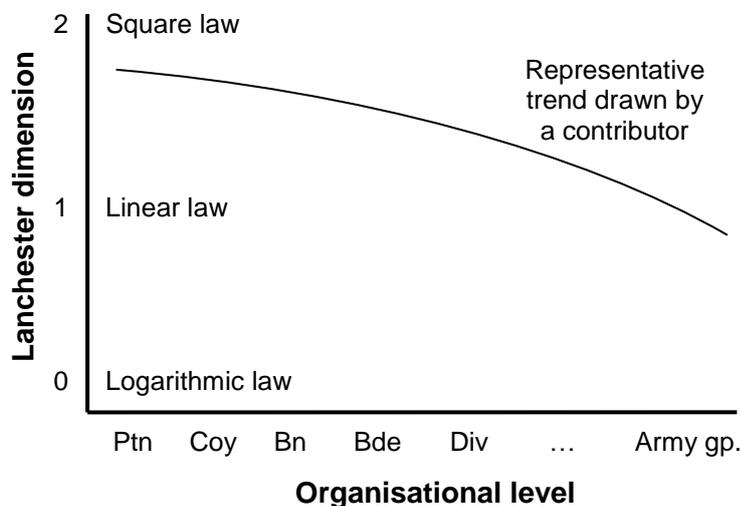


Figure 1. Probable variation in Lanchester exponent across scales of conflict

The reasons for this effect were a matter for speculation. First is that as the organisational level is raised, the balance between DF and artillery shifts towards the latter, so that the relative contribution of those weapons that deliver individual fire decreases. Secondly, as the organisational level is raised, the proportion of the troops on both sides that are actually engaged decreases, and particularly the engaged proportion of troops that operate DF weapons. (Corps artillery, assuming that it has the reach, will always be able to engage.)

The effects on combat are apparent in operational doctrine. It was Soviet Russian practice to attack in waves or echelons (e.g. see Isby, 1988), which would have disadvantaged them had combat at that level followed Lanchester's square law. Similarly, organising a defence in depth as successive lines of resistance exploits the same effect.

Asymmetric Lanchester laws: The workshop raised the possibility of an asymmetric model in which, for example, the defenders shot at the attackers using the square law, and the attackers returned fire on the defenders using the linear law.² Of course this happens naturally in an explicit combat model, where the defender might be using DF weapons and the attacker's main firepower might be provided by artillery, but it has not generally been considered by studies that sought to validate one particular Lanchester formulation against HA. (Bracken's 1995 general law quoted in Section 1 has two parameters, p and q , that apply equally to both sides; an asymmetric law would require four independent parameters.)

Minority weapon types: The workshop noted that some weapon types might need to be flagged as exceptions. **Snipers**, while they are anti-personnel weapons, are also point-to-point attackers, so that a shot that misses an intended target is highly unlikely to hit another. The

² Not surprisingly, this had already been discussed in the literature, in the context of counter-insurgency warfare, where different Lanchester equations were applied to each side; e.g. see Clausen (2003); Kress and Szechtman (2009).

same may apply to **precision guided munitions**, which are increasingly being fired by artillery in preference to unguided munitions (to minimise collateral damage).

The workshop briefly mentioned **landmines**, but did not dwell on their Lanchester effects. On later reflection, those that also aim to achieve area effects (e.g. the WW2 German *Schrapnell-Mine* and the US 'Bouncing Betty') might require a special case of the Lanchester equations, since they are both victim-initiated, thus sensitive to target force density, and the casualties inflicted are *again* proportional to target density after initiation. This implies a new Lanchester equation of the form,

$$dB/dt = -kb.R.B^2, \quad dR/dt = -kr.B.R^2.$$

While this could be seen as a case of Bracken's general law where $p = 1$ and $q = 2$, in practice it is unusual for either p or q to be greater than 1.0.

To complicate matters further, mines are persistent. This implies a time dimension, since a minefield is 'available' to attack an enemy force through the entire battle, unlike an artillery mission, which only attacks targets when it is fired. It is also unique in Lanchester terms, in that it is possible for a target to be killed by an enemy that has itself already been killed.

3. How do we implement this in combat models?

This question aimed to have the workshop consider how constructive combat models, including both wargames and simulations, might be modified in detail in order that they would be more likely to achieve a 'Lanchester linear effect' with force ratio.

Dismounted combat: There was agreement that if dismounted combat follows Lanchester's linear law, then the first change to most existing models must be to simulate DF anti-personnel weapons (with the probable exception of snipers) as bursting munitions, i.e. treating them as if they were grenade launchers (as argued by Lepingwell, 1987).³

More emphasis should be placed on modelling **suppressive fire**, and particularly on its feed-back into attrition rates (i.e. the suppressed units no longer contribute to the attrition, but by being suppressed they are of the order of 100 times less vulnerable per unit time). A few well-placed weapons can suppress a large enemy force, removing them from the 'combat active fraction'. In the discussion following Dr. John Salt's paper on modelling suppression (given later at the same symposium) it was opined that most combat models over-estimate the speed with which suppressed troops can recover, and return to contributing to the battle.

Allied to this is the modelling of **rate of fire** by all DF weapons, but particularly by small-arms. It was pointed out in the workshop that few infantry carry enough ammunition for more than five minutes' fire at their maximum rate. Many models assume that troops will always fire at their maximum rate, in order to maximise their attrition rate, whereas in reality much (even most?) fire is speculative and at a lower rate, and once a target is suppressed, it takes a lot less fire – and sometimes only the threat of continued fire – to keep them suppressed. Improving the rules regarding the selection of the rate of fire will improve the modelling of suppression, and of realistic close combat attrition rates in general.

Terrain effects: Good terrain modelling is important for getting the force densities right, both on attacker's approach routes and the ability of the defence to cover them. This in turn will

³ Note that this is not required in models such as Dstl's Close Action Environment (CAEn), where the model already takes account of bullet trajectories, implicitly producing a Lanchester linear effect.

influence the attrition – if the nature of the combat is Lanchester linear. Line-of-sight (LoS) opportunities must be modelled realistically as they affect the optimum balance between direct and indirect weapons. Models must also allow the **microterrain** to affect movement, and the extent to which it reduces the vulnerability of suppressed personnel.

Frontage effects: Lanchester models need to consider what proportion of each side's strength is engaged at any one time. Consideration of attack frontage is one of the main ways in which defending units can be excluded from participation, and if there is no explicit representation of space, then the 'square root law' of frontages offers a potential mechanism to implement this. In constructive combat simulations this already happens explicitly.

ISTAR effects: In crude Lanchester square models that include an element of STA, it is often treated as a straight multiplier, along with P(LoS), on the attrition rate. A better model would be to treat the probability of each firer acquires each potential target group as being independent, so that if more potential targets are presented, the number of shots taken will increase. In constructive combat simulations this already happens explicitly.

Low-level Lanchester models need to consider which side fires first, as this creates a huge advantage in that particular 'micro-battle'. Constructive combat models that represent ISTAR explicitly will already allow for this by default.

Model validation: The workshop agreed that it was important to validate models *across the feasible range of force ratios and at different force scalings*,⁴ in order to assess how their 'Lanchester signatures' (illustrated in Figure 1) compare with the results of HA. Having done this, we would be in a position to see what corrective action (if any) would be necessary.

Manual wargames: A later side discussion at ISMOR considered how a Lanchester linear effect might be obtained in a manual tactical wargame, such as the rules used in many Dstl studies.⁵ The main suggestion was that unlike many tactical games (e.g. SPI's *Firefight* model by Dunnigan & Hardy, 1976), infantry firepower should be applied to target *hexes*, not individual target units, as it was in *Soldiers* (by Isby, 1972) and in *Squad Leader* (by Hill, 1977). This can be amplified by relaxing the stacking limits (within reason), but most importantly, by implementing command and control rules that allow players to move a limited number of stacks per turn. In this way players are forced to choose between concentration, allowing them to move more of their force simultaneously to act in concert, and dispersion, which reduces casualties, but makes concerted action much harder (discussed by English & Gudmundsson, 1994).

4. How do we prepare the new data?

If a model of ground manoeuvre combat were to adopt a Lanchester linear model of infantry combat, the data would clearly need to change from being based on single shot kill probabilities (SSKPs) to the equivalent of mean areas of effect (MAEs) for artillery, presumably based on the beaten zones of the weapons under any particular set of conditions. In the event, it was difficult to discuss this topic in the workshop when there was no consensus on how a new model might

⁴ Clearly a combat model can only be tested within limited scaling range; thus it would be sensible, for example, to test a company close combat model at platoon and company force levels, potentially with a stretch to battalion or battlegroup, but it would be folly to attempt a brigade scenario, or to expect sensible results from modelling a section firefight.

⁵ A prime example of this was the *Exercise Kestrel's Hover* air manoeuvre wargame that was demonstrated by Mike Young at 32 ISMOR. The dismounted ground combat resolution mechanism in this game was deliberately designed to be Lanchester linear.

work. There was however agreement that whatever new models might be developed, they would need to take account of the feasibility of providing them with data.

A consequence of the scaling effects described in Section 2, that the Lanchester exponents change across organisational levels, is that HA validation studies need to pay more attention to the dimensions of combat, rather than treating it as a scale-invariant phenomenon. This in turn implies that future HA databases need to include more battles at the lower end of combat (i.e. at company and battalion) to have sufficient data points to analyse for the variables that are already known to affect outcome (e.g. technology levels, training, numbers, shock and surprise, terrain, and defensive preparations).

5. What are the implications for getting this wrong?

The obvious implication, which had also been discussed at length by Handel (1981), Kirkpatrick (1985), and Lepingwell (1987), was in giving advice on the **balance between quality and quantity** in military procurement and force balancing. A Lanchester square view of the world favours 'hordes' of individually less capable platforms or units, while a Lanchester linear view weights quality and quantity equally. It is interesting to note that during the Cold War, when most combat modellers were promoting Lanchester square models, that NATO generally adopted a policy of 'quality before quantity', because of the dominant costs of manpower in countries with a free market economy. Similarly, attacking in echelon and defence in depth make little sense if combat follows the square law, but make good tactical sense if combat follows the linear law.

More subtle are the implications for **concentration versus dispersion** in military doctrine and tactics, and as a consequence the value of investing in **command, control and communications (C3)**, because these are more critical in coordinating a dispersed force, if it is to bring force to bear locally and temporarily for decisive action, while relying on dispersion at other times for force protection. C3 advantages can thus amplify attrition advantages.⁶ (The extent to which C3 advantages can compensate for attrition deficits is another matter.)

The ability to concentrate and disperse forces for concerted action and for survivability also depends on their **ability to manoeuvre** across the terrain, be this on account of their own capabilities (boots, tracks, or being amphibious) or on account of having sufficient enablers available (e.g. bridging, heavy equipment transporters, or helicopters).

It must be emphasised that this workshop had focussed on ground combat, and that it **might be different for naval and air combat**. Technological differences may be even more salient in these environments, where for example having a significant range advantage might allow one force to destroy another for no loss, regardless of relative numbers (within reason).

If OR advice were consistently *and demonstrably* wrong in a conflict setting, then poor modelling would contribute to a **poor reputation for OR**, which would limit the future benefits that it could potentially deliver.

⁶ A participant reminded the workshop that J.C. Fuller had stated that the key to Lanchester was the concentration of *firepower*, not necessarily the concentration of troops.

6. Present author's conclusions

The first attempt to validate Lanchester models against observable reality was by Engel in 1954, and the first real indication as to what this debate might conclude was by Willard in 1962 (see also the review by Syms, 2017). Thus it is disappointing, sixty years on, to see how little influence that HA has had on constructive battle modelling, given that it is largely the latter that is used to advise on policy and operational decisions. This is not a new observation – the US Government Accountability Office (GAO) report of 1980 concluded that, “From a scientific point of view, the present understanding of war is in a relatively primitive state.” Davis and Blumenthal, writing in 1991, believed that combat modelling had still not progressed materially, and that this was due to the lack of a solid foundation in understanding warfare. For various reasons, among which (the present author speculates) are inertia, commercial pressures, sunk costs, and a lack of awareness of these matters, little has changed since then.

The lack of awareness in particular arises in part from the gulf between three separate analytical communities: the mathematical community, epitomised by Taylor (1980), who are interested in deriving equations to represent different combat situations; the HA community, epitomised by Rowland (1987), who are interested in extracting what data they can from history, and explaining the messy trends they uncover; and finally the constructive battle modelling community of both wargamers and simulationists, who build their models from the bottom up, spending little effort on validating the emergent behaviours of their models against the findings of either of the other two communities.

The present author hopes that at the least, the workshop and the related paper will have raised the profile of the issue a little from the obscurity to which it had sunk, so that when combat modellers embark on designing a new model, or refurbishing an existing one, they will have some ideas and pointers as to how to design one that more closely mirrors reality.

Acknowledgement

The author sincerely thanks those who attended the workshop, and who contributed to such a lively discussion, both at the time, and in the bar afterwards. Without them ISMOR would have been a duller and far less productive symposium.

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Topics

- Introduction: Lanchester's models (refresher)
- How does this work in reality?
- How do we implement this in combat models?
- How do we gather and prepare the new data?
- What are the implications for getting this wrong?

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Lanchester's original two laws

- Lanchester's 'square law' for individual fire

$$dB/dt = -kb.R, \quad dR/dt = -kr.B$$

– force effectiveness = individual effectiveness $\times n^2$

- Lanchester's 'linear law' for area fire

$$dB/dt = -kb.R.B, \quad dR/dt = -kr.B.R$$

– force effectiveness = individual effectiveness $\times n$

- *The weight of evidence favours using the linear law for most modern combat*



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Extensions of Lanchester's laws

- Mixed law: Smith (1965)
 - sum components of linear and square laws

- Logarithmic law: Peterson (1967)

$$dB/dt = -kb.B, \quad dR/dt = -kr.R$$

– loss rates determined by *own* numbers only, e.g. DNBIs, FF

- General law: Bracken (1995)

$$dB/dt = -kb.R^p.B^q, \quad dR/dt = -kr.B^p.R^q$$



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Time plan (tentative!)

- 11:00–11:20 Lanchester's models (refresh)
- 11:20–11:30 How does this work in reality?
- 11:30–11:40 How do we do this in combat models?
- 11:40–11:50 How do we prepare the new data?
- 11:50–12:00 Implications for getting this wrong?

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